[This question paper contains 4 printed pages.]

Your Roll No..............

Sr. No. of Question Paper : 2915 **H**

Unique Paper Code : 32351201

Name of the Paper : Real Analysis (CBCS-LOCF)

Name of the Course : **B.Sc. (Hons) Mathematics**

Semester : II

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.

2. **All** questions are compulsory.

3. Attempt any **two** parts from each question.

1. (a) Let S be a nonempty bounded set in $\mathbb{R}$. Let $b < 0$, and let $bS = \{bs: s \in S]$. Prove that $\inf(bS) = b(\sup S)$ and $\sup(bS) = b(\inf S)$. (6.5)

(b) State and prove Archimedean Property of real numbers. (6.5)

P.T.O.

(c) Prove that (0,1] is neither an open set nor a closed set. (6.5)

2. (a) If a, b $\in \mathbb{R}$, then prove that $||a| - |b|| \leq |a - b| \leq |a| + |b|$. (6)

(b) Prove that an upper bound u of a nonempty set S in $\mathbb{R}$ is the supremum of S if and only if for every $\varepsilon > 0$ there exists an $s_\varepsilon \in S$ such that $u - \varepsilon < s_\varepsilon$. (6)

(c) If S is nonempty subset of $\mathbb{R}$, show that S is bounded if and only if there exists a closed and bounded interval I such that $S \subseteq I$. (6)

3. (a) Prove that $\lim_{n\to\infty} x_n = 0$ if and only if $\lim_{n\to\infty} |x_n| = 0$. Give an example to show that the convergence of $\langle |x_n| \rangle$ need not imply convergence of $\langle x_n \rangle$. (6.5)

(b) Prove that every monotonically increasing bounded above sequence is convergent. (6.5)

(c) Define Cauchy sequence. Show directly using the definition that

$$\left\langle 1+\frac{1}{2!}+\cdots\cdots+\frac{1}{n!}\right\rangle$$ is a Cauchy sequence. (6.5)

4. (a) If $\langle a_n \rangle$ and $\langle b_n \rangle$ converges to a and b respectively, prove that $\langle a_n + b_n \rangle$ converges to $(a+b)$. (6)

(b) Show that a sequence in $\mathbb{R}$ can have at the most one limit. (6)

(c) Show that $\lim_{n\to\infty} c^{1/n} = 1$ for $c > 0$. (6)

5. (a) Examine the convergence or divergence of the following series.

(i) $\frac{2}{3}+\frac{3}{4}+\frac{4}{5}+\frac{5}{6}+\frac{6}{7}+\cdots$

(ii) $\frac{\sqrt{2}}{3.5}+\frac{\sqrt{4}}{5.7}+\frac{\sqrt{6}}{7.9}+\frac{\sqrt{8}}{9.11}+\frac{\sqrt{10}}{11.13}+\cdots$ (6.5)

(b) Prove that $\sum_{n=1}^{\infty}\frac{1}{n^2+n}$ is a telescoping series, and find its sum. (6.5)

(c) Prove that the p-series $\sum_{n=1}^{\infty}\frac{1}{n^p}$, $p>0$ converges for $p>1$ and diverges for $p \le 1$. (6.5)

6. (a) State and prove ratio test (limit form). (6)

(b) Examine the convergence of the following series, clearly stating the test used

$$\frac{1}{e}+\frac{4}{e^2}+\frac{27}{e^3}+\frac{256}{e^4}+\frac{3125}{e^5}+\cdots \quad (6)$$

(c) Define Alternating series of real numbers. Test for the absolute convergence and convergence of the series $1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+\cdots$ (6)

[This question paper contains 4 printed pages.]

Your Roll No..............

Sr. No. of Question Paper : 3005 H

Unique Paper Code : 32351402

Name of the Paper : Riemann Integration and Series of Functions

Name of the Course : B.Sc. (Hons.) Mathematics

Semester : IV

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. **All** questions are compulsory.
3. Attempt any **two** parts from each question.
4. Use of calculator is not allowed.

1. (a) Find the upper and lower Darboux integral for $f(x) = 2x + 1$ and show that it is integrable on $[1,2]$. Hence show that $\int_1^2 (2x + 1)dx = 4$. 6.5)

   (b) Prove that a bounded function f on [a, b] is Riemann integrable if and only if it is Darboux integrable in which case the values of the integrals agree. (6.5)

(c) Prove that every continuous function f on [a, b] is integrable. (6.5)

2. (a) (i) State and prove intermediate value theorem for integrals.

(ii) Give example of a function f which is not integrable for which |f| is integrable. (6)

(b) If u and v are continuous functions on [a, b] that are differentiable on [a, b] and if u′ and v′ are integrable on [a, b] then prove

$$\int_a^b u(x)v'(x)+\int_a^b u'(x)v(x)=u(b)v(b)-u(a)v(a)$$

(6)

(c) Prove that if f is a piecewise continuous function or a bounded piecewise monotonic function on [a, b] then f is integrable on [a, b]. (6)

3. (a) Prove that $\int_0^\infty e^{-t}t^{s-1}dt$ is convergent if and only if $s > 0$. (6)

(b) (i) Prove that $\int_1^\infty \frac{\sin}{x^2}\,dx$ converges absolutely

(ii) Prove that $\int_1^{\infty} \frac{\sqrt{x}}{x^3+5} dx$ is convergent. (6)

(c) Test convergence of

(i) $\int_2^{\infty} \frac{2x^2}{x^4-1} dx$ (ii) $\int_0^{\infty} e^{-x^2} x^2 dx$ (6)

4. (a) Let $\langle f_n \rangle$ be a sequence of bounded functions on $A \subseteq R$. Then this sequence converges uniformly on A to a bounded function f if and only if for each $\in > 0$ there is a number $H(\in)$ in N such that for all $m, n \geq H(\in)$ then $||f_m - f_n||_A \leq \in$. (6.5)

(b) Let $f_n(x) = \frac{nx}{1+n^2x^2}$ for $x \geq 0$, $n \in N$. Show that the sequence $\langle f_n \rangle$ converges non-uniformly on $[0, \infty)$ and converges uniformly on $[a, \infty)$, $a > 0$. (6.5)

(c) If $a > 0$, show that $\lim_{n\to\infty} \left(\int_a^{\pi} \frac{\text{Sin } nx}{nx} dx \right) = 0$. What happens if $a = 0$. (6.5)

5. (a) State and prove Weierstrass M-test for the uniform Convergence of a series of functions. (6.5)

(b) Discuss the convergence and uniform convergence of the series of functions

$$\Sigma(x^n + 1)^{-1}, \quad x \neq 0 \qquad (6.5)$$

(c) If f_n is continuous on $D \subseteq R$ to R for each $n \in N$ and Σf_n converges to f uniformly on D then show that f is continuous on D. (6.5)

6. (a) (i) Find the radius of convergence of the power series (3)

$$\sum_{n=1}^{\infty} \frac{n^n}{n!}$$

(ii) Define sinx as a power series and find its radius of convergence. (3)

(b) If the power series $\sum_{n=0}^{\infty} a_n x^n$ has radius of convergence R, then the power series

$\sum_{n=1}^{\infty} n a_n x^{n-1}$ and $\sum_{n=0}^{\infty} \frac{a_n}{n+1} x^{n+1}$ also have radius of convergence R. (6)

(c) Suppose that $f(x) = \sum_{n=0}^{\infty} a_n x^n$ has radius of convergence $R > 0$. Then f is differentiable on $(-R, R)$ and

$$f'(x) = \sum_{n=1}^{\infty} n a_n x^{n-1} \text{ for } |x| < R. \qquad (6)$$

[This question paper contains 4 printed pages.]

Your Roll No..............

Sr. No. of Question Paper : 3116 **H**

Unique Paper Code : 32351403

Name of the Paper : Ring Theory & Linear Algebra-I

Name of the Course : **B.Sc. (Hons.) Mathematics CBCS (LOCF)**

Semester : IV

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. **All** questions are compulsory.
3. Attempt any **two** parts from each question.

1. (a) Prove that every finite Integral domain is a field. Give an example of an infinite integral domain which is not a field, Justify. (6)

   (b) (i) Let F be a field of order 2^n. Prove that the characteristic of F is 2.

   (ii) Find all the units in $\mathbb{Z}[i]$ (6)

   (c) Prove that the set of all the nilpotent elements of a commutative ring form a subring. (6)

P.T.O.

2. (a) Let R be a commutative ring with unity and A be an ideal of R then prove that R/A is an integral domain if and only if A is a prime ideal of R. (6)

(b) Prove that $\mathbb{Z}[i]/\langle 1-i\rangle$ is a field. (6)

(c) Find all the maximal ideals of $\mathbb{Z}_{20}$. (6)

3. (a) Find all the ring homomorphisms from $\mathbb{Z}_4$ to $\mathbb{Z}_{10}$. (6.5)

(b) Let $R=\left\{\begin{bmatrix} a & b \\ 0 & c \end{bmatrix} \middle| a,b,c\in\mathbb{Z}\right\}$ and Φ be the mapping that takes $\begin{bmatrix} a & b \\ 0 & c \end{bmatrix}$ to a. Show that

(i) Φ is a ring homomorphism

(ii) Determine the kernel of Φ.

(iii) Is Φ a one-one mapping. Justify. (6.5)

(c) State and prove first isomorphism theorem for rings. (6.5)

4. (a) Let V(F) be a vector space.

(i) Prove that the intersection of two subspaces of V(F) is also a subspace of V(F).

(ii) Show that union of two subspaces of V(F) may not be a subspace of V(F). Discuss the condition under which union of two subspaces will also form a subspace of V(F). (6)

(b) Let S be a linearly independent subset of a vector space V(F), and let v be a vector in V that is not in S. Then $S \cup \{v\}$ is linearly dependent iff $v \in \text{Span}(S)$. (6)

(c) Let u, v, w be distinct vectors of a vector space V. Show that if {u,v,w} is a basis for V, then {u+v+w, v+w, w} is also a basis for V. (6)

5. (a) Let V(F) and W(F) be vector spaces and let $T: V \to W$ be a linear transformation. If V is a finite-dimensional, then

$$\text{Dim}(V) = \text{Nullity}(T) + \text{Rank}(T).$$ (6.5)

(b) Let $T: \mathbb{R}^2 \to \mathbb{R}^3$ and $U: \mathbb{R}^2 \to \mathbb{R}^3$ be the linear transformations respectively defined by

$T(a_1, a_2) = (a_1 + 3a_2, 0, 2a_1 - 4a_2)$

$U(a_1, a_2) = (a_1 - a_2, 2a_1, 3a_1 + 2a_2)$

Let β, γ be the standard basis of $\mathbb{R}^2$ and $\mathbb{R}^3$ respectively, Prove that

(i) $[T+U]_\beta^\gamma + [T]_\beta^\gamma + [U]_\beta^\gamma$

(ii) $[aT]_\beta^\gamma = a[T]_\beta^\gamma$ for all scalars a. (6.5)

(c) Let V and W be vector spaces. Let $T: V \rightarrow W$ be linear and let $\{w_1, w_2, \ldots, w_k\}$ be a linearly independent subset of Range of T. Prove that if $S = \{v_1, v_2, \ldots v_k\}$ is chosen so that $T(v_i) = w_i$ for $i = 1, \ldots, k$. Then S is linearly independent.

(6.5)

6. (a) Let T be a linear operator on a finite dimensional vector space V. Let β and β' be the ordered basis for V. Suppose that Q is the change of coordinate matrix that changes β' coordinates into β coordinates, then $[T]_{\beta'} = Q^{-1}[T]_\beta Q$. (6.5)

(b) Let β, γ be the standard ordered basis of $P_1(\mathbb{R})$ and $\mathbb{R}^2$ respectively.

Let $T: P_1(\mathbb{R}) \rightarrow \mathbb{R}^2$ be a linear transformation defined by $T(a + bx) = (a, a+b)$.

Find $[T]_\beta^\gamma$, $\left[T^{-1}\right]_\gamma^\beta$ and verify that $\left[T^{-1}\right]_\gamma^\beta = \left([T]_\beta^\gamma\right)^{-1}$.

(6.5)

(c) Let $U: P_3(\mathbb{R}) \rightarrow P_2(\mathbb{R})$ and $T: P_2(\mathbb{R}) \rightarrow P_3(\mathbb{R})$ be the linear transformations respectively defined by

$U(f(x)) = f'(x)$ and $T(f(x)) = \int_0^x f(t)\,dt$. Prove that $[UT]_\beta = [U]_\alpha^\beta [T]_\beta^\alpha$ where α and β are standard ordered basis of $P_3(\mathbb{R})$ and $P_2(\mathbb{R})$ respectively.

(6.5)

[This question paper contains 8 printed pages.]

Your Roll No..............

Sr. No. of Question Paper : 4064 **H**

Unique Paper Code : 2352012401

Name of the Paper : Sequence and Series of Functions

Name of the Course : **B.Sc. (Hons.) Mathematics**

Semester : IV

Duration : 3 Hours Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.

2. Attempt any **two** parts from each question.

3. **All** questions are compulsory and carry equal marks.

1. (a) Define uniform convergence of a sequence of functions (f_n) defined on $A \subseteq \mathbb{R}$ to $\mathbb{R}$. If $A \subseteq \mathbb{R}$ and $\varphi: A \to \mathbb{R}$ then define the uniform norm of

P.T.O.

φ on A. Discuss the uniform and pointwise convergence of the sequence (f_n), where

$$f_n(x) = \frac{x}{n} \text{ for } x \in \mathbb{R} \text{ and } n \in \mathbb{N}.$$

(b) Let (f_n) be the sequence of functions defined by

$$f_n(x) = \frac{1}{1+x^n} \quad \forall x \in [0,1],\ n \in \mathbb{N}.$$

Find the pointwise limit of the sequence (f_n). Does (f_n) converges uniformly? Justify your answer.

(c) Show that if (f_n) and (g_n) are two sequences of bounded functions on $A \subseteq \mathbb{R}$ to $\mathbb{R}$ that converge uniformly to f and g respectively then prove that the product sequence $(f_n g_n)$ converge uniformly on A to fg. Give an example to show that in general the product of two uniformly convergent sequence may not be uniformly convergent.

2. (a) Let (f_n) be a sequence of integrable functions on $[a, b]$ and suppose that (f_n) converges uniformly to f on $[a, b]$. Show that f is integrable on $[a; b]$ and

$$\int_a^b f = \lim_{n\to\infty} \int_a^b f_n$$

(b) Let $f_n(x) = \frac{x^n}{n}$ for $x \in [0, 1]$. Show that the sequence (f_n) of differentiable functions converges uniformly to a differentiable function f on $[0, 1]$ and that the sequence (f_n') converges on $[0, 1]$ to a function g but the convergence is not uniform.

(c) Show that the sequence $\left(\frac{x^n}{1+x^n}\right)$ does not converge uniformly on [0,2].

3. (a) State and prove Weierstrass M-Test for uniform convergence of series of functions.

(b) Show that the series $\sum_{n=1}^{\infty} \sin\left(\frac{x}{n^2}\right)$ is uniformly convergent on [–a, a], a > 0 but is not uniformly convergent on $\mathbb{R}$.

(c) Discuss the pointwise convergence of the series of functions $\sum_{n=1}^{\infty} \frac{x^n}{2+3x^n}$ for $x \geq 0$.

4. (a) Let f_n be continuous function on $D \subseteq \mathbb{R}$ to $\mathbb{R}$ for each $n \in N$ and $\sum_{n=1}^{\infty} f_n$ converges uniformly to f on D. Prove that f is continuous on D.

(b) Show that the series of functions $\sum_{n=1}^{\infty} \frac{\cos(x^2+1)}{n^3}$ converges uniformly on $\mathbb{R}$ to a continuous function.

(c) Given the Riemann integrable functions $f_n: \mathbb{R} \to \mathbb{R}$, $n \in N$, such that

$$f_n(x) = \sin\left(\frac{x}{n^4}\right) \text{ for all } x \in \mathbb{R}$$

Show that $\sum_{n=1}^{\infty} \int_{-2\pi}^{2\pi} f_n(x)\,dx = \int_{-2\pi}^{2\pi} \sum_{n=1}^{\infty} f_n(x)\,dx$.

5. (a) Define the radius of convergence and interval of convergence of power series. Check the uniform convergence of the following power series on [−1, 1]

$$x+\frac{x^2}{2^2}+\frac{x^3}{3^2}+\frac{x^4}{4^2}+\ldots\ldots\ldots\ldots.$$

(b) State and Prove Cauchy-Hadamard Theorem.

(c) Suppose that $f(x)=\sum_{n=0}^{\infty} a_n x^n$ has radius of convergence $R>0$. Then prove that the series $\sum_{n=0}^{\infty}\frac{a_n}{n+1}x^{n+1}$ has also radius of convergence R and for $|x|<R$

$$\int_0^x f(t)dt = \sum_{n=0}^{\infty}\frac{a_n}{n+1}x^{n+1}.$$

6. (a) For cosine function $C(x)$ and sine function $S(x)$ prove the following :

(i) If $f: \mathbb{R} \to \mathbb{R}$ is such that $f''(x) = -f(x)$ for $x \in \mathbb{R}$ then there exist real numbers α and β such that $f(x) = \alpha C(x) + \beta S(x)$ for $x \in \mathbb{R}$.

(ii) If $x \in \mathbb{R}$, $x \geq 0$ then

$$1-\frac{1}{2}x^2 \leq C(x) \leq 1-\frac{1}{2}x^2+\frac{1}{24}x^4.$$

(b) State Abel's Theorem. Show that

$$\tan^{-1}x = x-\frac{x^3}{3}+\frac{x^5}{5}-\frac{x^7}{7} \ldots\ldots\ldots\ldots -1 \leq x \leq 1$$

and $\frac{\pi}{4}=1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+\ldots\ldots\ldots\ldots$

(c) Prove that for every continuous function f on [0, 1], the sequence of polynomials $B_n f \to f$ uniformly on [0, 1], where $(B_n f)$ is the sequence of Bernstein's polynomials for the function f.

[This question paper contains 4 printed pages.]

Your Roll No...............

H

Sr. No. of Question Paper : 2933

Unique Paper Code : 32351601

Name of the Paper : Complex Analysis (including Practicals)

Name of the Course : **B.Sc. (H) Mathematics**

Semester : VI

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. **All** questions are compulsory.
3. Each question consists of **three** parts. Attempt any **two** parts from each question.

1. (a) (i) Find and sketch, showing corresponding orientations, the image of the hyperbola $x^2 - y^2 = c_1$ $(c_1 < 0)$ under the transformation $w = z^2$.

(ii) Suppose $(z)=\begin{cases} z/|z| & \text{if } z \neq 0 \\ 0 & \text{if } z=0 \end{cases}$. Check the continuity of f at z = 0. Justify your answer.

(4+2=6)

P.T.O.

(b) (i) Determine the points where $f(z) = (x^3 + 3xy^2 - x) + i(y^3 + 3x^2y - y)$ is differentiable. Is f analytic at those points? Justify your answer.

(ii) Suppose f(z) is analytic. Can $g(z)=\overline{f(z)}$ be analytic? Justify your answer. (4+2=6)

(c) (i) Suppose $f(z) = e^{-x}e^{-iy}$. Show that $f'(z)$ and $f''(z)$ exist everywhere. Hence prove that $f''(z) = f(z)$.

(ii) Show that $\log(i^{1/2}) = (1/2) \log i$.

(3.5+2.5=6)

2. (a) (i) Find all the roots of the equation $\sin z = 5$.

(ii) Show that $|\exp(z^2)| \leq \exp(|z^2|)$. (4+2=6)

(b) (i) Show that $\sin \bar{z}$ is not analytic function anywhere.

(ii) Compute $\lim\limits_{z\to\infty} \dfrac{2z^3-1}{z^2+1}$. (5+1=6)

(c) Evaluate $\int_C \frac{dz}{z}$, where C is a positively oriented circle $z = 2e^{i\theta}$ $(-\pi \leq \theta \leq \pi)$. (6)

3. (a) Let $f(z) = \pi \exp(\pi\bar{z})$ and C is the boundary of the square with vertices at the points $0, 1, 1+i$ and i; orientation of C being in the anticlockwise direction. Parametrize the curve C and evaluate $\int_C f(z)dz$. (6)

(b) Let C denote a contour of length L and suppose that a function f(z) is piecewise continuous on C. If M is a non-negative constant such that $|f(z)| \leq M$ for all points z on C at which f(z) is defined, then prove that $\left|\int_C f(z)dz\right| \leq ML$. (6)

(c) If a function f is analytic at a given point, then prove that its derivatives of all orders are analytic there too. (6)

4. (a) State Cauchy integral formula and its extension. Evaluate $\oint_C \frac{z+1}{z^4 + 2iz^3} dz$, where C is the circle $|z| = 1$. (6)

(b) Show that if f is analytic within and on a simple closed contour C and z_0 is not on C, then

$$\int_C \frac{f'(z)}{z - z_0} dz = \int_C \frac{f(z)}{(z - z_0)^2} dz$$ (6)

(c) State and prove Fundamental theorem of Algebra. (6)

5. (a) Find limit of the following sequences as $n \to \infty$:

(i) $z_n = \frac{1}{n^3} + i \quad (n = 1,2,....)$

(ii) $w_n = -2 + i\frac{(-1)^n}{n^2} \quad (n = 1,2,...)$ (3+3.5)

(b) Represent the function $f(z)=\frac{z+1}{z-1}$.

(i) by its Maclaurin series and state the domain where the representation is valid.

(ii) by its Laurent series in the domain $1 < |z| < \infty$. (3+3.5)

(c) Find the residue at $z = 0$ of the functions :

(i) $\frac{1}{z+z^2}$ (ii) $z\cos(1/z)$ (3+3.5)

6. (a) State Cauchy Residue Theorem and use it to evaluate the integral of $\frac{1}{1+z^2}$ around the circle $|z| = 2$ in the positive sense. (7)

(b) Show that the point $z = 0$ is a pole of the function $z(e^z - 1)$. Also find order of the pole and corresponding residue. (7)

(c) In each case, write the principal part of the function at its isolated singular point and determine the type of singular point with full explanation :

(i) $e^{1/z}$ (ii) $\frac{z^2}{1+z}$ (3.5+3.5)

[This question paper contains 8 printed pages.]

Your Roll No..............

Sr. No. of Question Paper : 3070 **H**

Unique Paper Code : 32357609

Name of the Paper : Bio-Mathematics

Name of the Course : **B.Sc. (H) Mathematics**

Semester : VI

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. All the **six** questions are compulsory.
3. Attempt any **two** parts from each question.
4. Use of Scientific Calculator is allowed.

1. (a) Observations on animal tumours indicate that their sizes obey the Gompertz growth law $\frac{ds}{dt} = ks\ln\left(\frac{B}{s}\right)$ rather than the logistic law. Here k and B are

P.T.O.

positive constants. By putting $y = \ln(s)$, prove that

$s(t) = Be^{-Ae^{-kt}}$, where $A = \ln\left(\frac{B}{s_0}\right)$, s_0 being the size at $t = 0$. Deduce that, in Gompertz growth, the size moves steadily from its initial value to an eventual value. (6)

(b) Determine the rest states in the Volterra-Lotka model of predator-prey interaction,

$$\frac{dX}{dt} = -eX + fXY, \quad \frac{dY}{dt} = aY - bY^2 - cXY.$$

Here, X is the predator density and Y the prey density. If predator and prey are present in steady state, show that the coefficients a, b, c, e and f must satisfy the constraints $\frac{a}{b} > \frac{e}{f}$, $c \neq 0$. (6)

(c) Describe the epidemic model and show that population returns to equilibrium after the small departure from the equilibrium. (6)

2. (a) Discuss the nature of equilibrium point and draw the trajectories in the phase plane for the following system :

$\dot{x} = x + 3y, \quad \dot{y} = -3x - y.$ (6.5)

(b) Transform the second order differential equation into first order system and examine its equilibrium points

$$\ddot{x} + 2b\dot{x} + ax = 0,$$

where a and b are constants and $a \neq 0$. (6.5)

(c) What is stable and unstable limit cycle? Give an example of a limit cycle. State limit cycle criterion and Poincare-Bendixon Theorem. What is the trapping region? (6.5)

3. (a) Describe a full phase plane analysis of the model

$$\in \frac{dx}{dt} = -\left(x^3 - Tx + b\right),\ T > 0$$

$$\frac{db}{dt} = \left(x - x_0\right) + \left(x_0 - x_1\right)u.$$

 P.T.O.

Here x represents muscle fibre length, u denotes the control variable for pacemaker, (b_0, x_0) is the rest state, b is for control variable, t is the time, x_1 is the systolic state and T denotes the tension. (6)

(b) Solve the ordinary differential equation

$$\frac{dy}{dt} = -\gamma y + u, \quad 0 < \gamma < \frac{1}{4}$$

to obtain the period T of periodic state y which characterizes the pacemaker. Assume that $0 \le y \le 1$, such that when y = 1, the pacemaker fires and y = 0, it jumps back. Here, it is the control variable. (6)

(c) Consider the system of differential equations

$$\dot{x} = \mu - x^2, \ \dot{y} = -y,$$

here μ is any real constant.

Show that this system possesses saddle-node bifurcation. (6)

4. (a) Show that the iteration scheme $x_{n+1} = 1 - \mu x_n(1 - x_n)$ has a stable fixed point $x_0 = 1$ for $\mu < 1$ and that $\mu = 1$ is a bifurcation point where the fixed point $x_0 = 1/\mu$ appears. Show that the period doubling occurs as soon as μ exceeds 3. (6.5)

(b) Show that

$$\dot{x} = \mu x(2y-1),$$

$$\dot{y} = \mu - y(2x+1),$$

has an equilibrium point, which is a stable node for $\frac{1}{2} < \mu < 1$ and a stable focus for $\mu > 1$. Prove that $\mu = \frac{1}{2}$ is a bifurcation point. (6.5)

(c) Define bifurcation and bifurcation point. Explain pitchfork bifurcation, saddle-node bifurcation and Hopf-bifurcation using diagrams. (6.5)

5. (a) Let $M(\alpha_1)$, $M(\alpha_2)$ be two Jukes-Cantor matrices with parameters α_1 and α_2 respectively. Show that their product is again a Jukes-Cantor matrix with parameter α_3. Also, give a formula for α_3 in terms of α_1 and α_2. (6)

(b) Define root and a bifurcating tree. Draw all the three topologically distinct unrooted bifurcating trees that could describe the relationship between 4 taxa. (6)

(c) If D and d denotes the alleles for tall and dwarf plant and if W and w denotes the alleles for round and wrinkled seed, then for the cross DdWw × ddWw compute the following probabilities :

(i) The probability of a tall plant with wrinkled seeds.

(ii) The probability of a tall plant with round seeds. (6)

6. (a) From the given distance table, construct a rooted tree showing the relationship between S1,S2,S3 and S4 by UPGMA.

	S1	S2	S3	S4
S1		1.2	0.9	1.7
S2			1.1	1.9
S3				1.6

(6.5)

(b) In mice, an allele A for agouti- or gray-brown, grizzled fur is dominant over the allele a, which determines a non-agouti color. If an Aa × Aa cross produces 6 offspring, then

(i) What is the probability that exactly 5 of 6 offspring have agouti fur?

(ii) What is the probability that more than half of 6 offspring have agouti fur?

(6.5)

(c) In pea plants, let Y denotes the dominant allele for yellow seed and y the recessive allele for green seeds, P denote the dominant allele for purple flowers and p the recessive allele for white flowers. Draw the Punnett square for the cross, YyPp × yyPp. Also, find the different genotypes and distinct phenotypes. (6.5)

[This question paper contains 8 printed pages.]

Your Roll No..............

Sr. No. of Question Paper : 3072 **H**

Unique Paper Code : 32357615

Name of the Paper : Introduction to Information Theory and Coding

Name of the Course : **B.Sc. (H) Mathematics**

Semester : VI

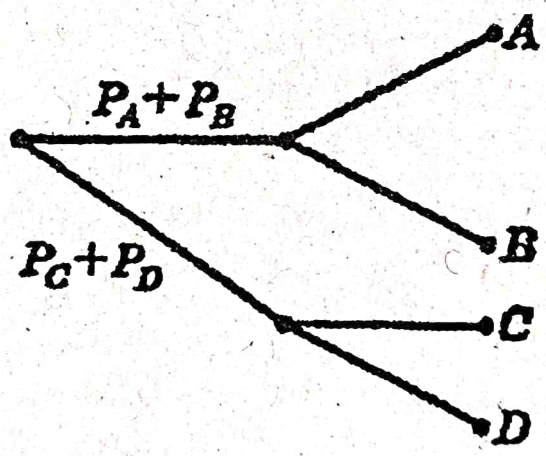
Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. This question paper consists of **six** questions in all.
3. **All** questions are compulsory.
4. Only **two** parts have to be attempted in each question.
5. **All** questions have an equal weightage.
6. Non-programmable scientific calculator and statistical tables are allowed.

1. (a) Define the entropy of a simple binary source. An alphabet consists of four letters A, B, C, D with respective probabilities of transmission 1/3, 1/4, 1/4, 1/6. Find the entropy associated with the transmission of a letter.

(b) Verify the rule of additivity of entropies for the following probability scheme



with input probabilities as

$$P(A)=\frac{1}{2},\ P(B)=\frac{1}{4},\ P(C)=\frac{1}{8},\ P(d)=\frac{1}{8}.$$

(c) Let $X_1, X_2, \ldots, X_n$ be drawn according to $p(x_1, x_2, \ldots, x_n)$. Then prove that

$$H(X_1, X_2, \ldots, X_n) = \sum_{i=1}^{n} H(X_i | X_{i-1}, X_{i-2}, \ldots, X_1).$$

2. (a) A transmitter has an alphabet consisting of five letters $\{x_1, x_2, x_3, x_4, x_5\}$ and the receiver has an alphabet of four letters $\{y_1, y_2, y_3, y_4\}$. The joint probabilities for the communication are given as :

$$\begin{array}{c|cccc} & y_1 & y_2 & y_3 & y_4 \\ \hline x_1 & 0.25 & 0 & 0 & 0 \\ x_2 & 0.10 & 0.30 & 0 & 0 \\ x_3 & 0 & 0.05 & 0.10 & 0 \\ x_4 & 0 & 0 & 0.05 & 0.10 \\ x_5 & 0 & 0 & 0.05 & 0 \end{array}$$

Verify the relation $H(X,Y) = H(Y) + H(X|Y)$.

(b) A binary channel has the following noise characteristic :

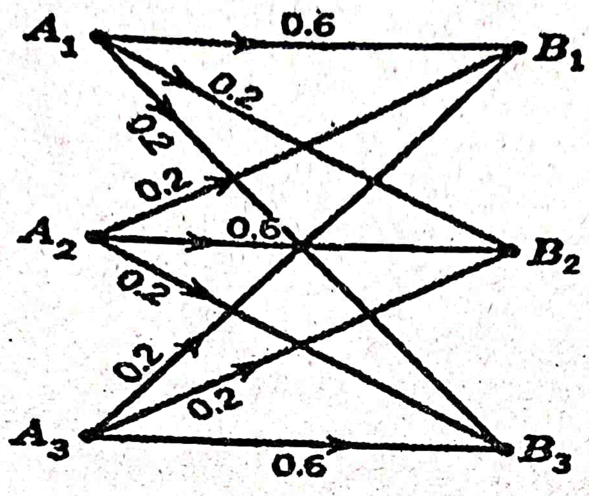
$$\begin{array}{c c} & \begin{array}{cc} 0 & 1 \end{array} \\ \begin{array}{c} 0 \\ 1 \end{array} & \begin{bmatrix} 2/3 & 1/3 \\ 1/3 & 2/3 \end{bmatrix} \end{array}$$

If the input symbols 0 and 1 are transmitted with respective probabilities of 3/4 and 1/4, verify the result:

$$H(X) - H(X|Y) = H(Y) - H(Y|X)$$

(c) Show that the entropy function $H(p_1, p_2, \ldots p_n)$ is continuous in each and every independent variable p_k in the interval $]0, 1]$.

3. (a) A discrete source transmits messages $[A_1, A_2, A_3]$ with respective probabilities [1/2, 1/4, 1/4]. The source is connected to the channel given as follows :



Determine H(A), H(B), I(A;B) and the channel capacity.

(b) Consider two random variables X and Y with a joint probability mass function p(x, y) and marginal probability mass functions p(x) and p(y). Define the mutual information I(X; Y) and prove that

$$I(X; Y) = H(X) + H(Y) - H(X, Y).$$

(c) Define the log sum inequality and prove that D(p||q) is convex in the pair (p, q).

4. (a) Describe a linear block code and clearly state the meaning of each parameter used there. Define the generator matrix G and the Parity check matrix H for a given linear block code. What is the relationship between G and H?

(b) Show that every generator matrix G of the dual code $C^{\perp}$ is a parity check matrix H of C.

(c) Define Binary symmetric channel. Let C be a binary [3, 1, 3] linear code with a Binary Symmetric Channel with cross-over probability p = 0.4. The decoder is defined as a function D: $\{0, 1\}^3 \rightarrow C$ such that D(001) = D(010) = D(100) = D(000) = 000 and D(011) = D(101) = D(110) = D(111) = 111.

Find P_{err} (the decoding error probability of the decoder D) up to 4 digits of decimal.

5. (a) Define syndrome for an n-tuple vector. For a linear code C, show that y_1 and y_2 are in the same row of the standard array for C if and only if syndromes of y_1 and y_2 are equal.

(b) Let C = [5, 2, 3] be a linear block code GF(2) with generator matrix $G = \begin{pmatrix} 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 \end{pmatrix}$.

Construct a standard array using the due procedure. Is standard array thus obtained unique?

(c) Find g.c.d.$(x^5 + x^4 + x^3 + x^2, x^2 + 1)$ by using Euclidean algorithm.

6. (a) State and prove the singleton bound theorem.

(b) Consider a linear code $(2^m - 1, k, 5)$ of minimum distance 5 over F = GF(2). What could be the maximum value of k? Determine.

(c) What are perfect codes? Consider the [n, n – m, 3] Hamming code over GF(q) where q > 1 and $n = (q^m - 1)/(q - 1)$. Is this code perfect?

[This question paper contains 8 printed pages.]

Your Roll No..............

Sr. No. of Question Paper : 2987 **H**

Unique Paper Code : 32351602

Name of the Paper : Ring Theory and Linear Algebra-II

Name of the Course : **B.Sc. (H) Mathematics (CBCS-LOCF)**

Semester : VI

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.

2. **All** the questions are compulsory.

3. Attempt any **two** parts from each question.

4. Marks of each part are indicated.

P.T.O.

1. (a) State and prove the Division Algorithm for $\mathbb{F}[x]$, where $\mathbb{F}$ is a field.

(b) State and prove Eisenstein's Criterion.

(c) Let $\mathbb{F}$ be a field and

$I = \{a_0 + a_1x + a_2x^2 + \cdots + a_nx^n | a_i \in \mathbb{F}$, $i = 0,1,2,\ldots, n$ and $\sum_{i=0}^{n} a_i = 0\}$. Show that I is an ideal of $\mathbb{F}[x]$ and find a generator of I.

(6,6,6)

2. (a) Let $\mathbb{F}$ be a field and $p(x) \in \mathbb{F}[x]$. Prove that $<p(x)>$, is a maximal ideal in $\mathbb{F}[x]$ if and only if p(x) is irreducible.

(b) Prove that every Euclidean Domain is a Principal Ideal Domain.

(c) In the ring $\mathbb{Z}[\sqrt{5}]$, show that the element $1+\sqrt{5}$ is irreducible but not prime. (6.5,6.5,6.5)

3. (a) Let $V = \mathbb{R}^3$ and define $f_1, f_2, f_3 \in V^*$ as follows :

$$f_1(x,y,z) = x - \frac{1}{2}y, \quad f_2(x,y,z) = \frac{1}{2}y, \quad \&$$

$$f_3(x,y,z) = -x + z$$

Prove that $\{f_1, f_2, f_3\}$ is a basis for V^* and then find a basis for V for which it is the dual basis.

(b) Let $V = P_3(\mathbb{R})$ and $T(f(x)) = f(x) + f(2)x$. Show that T is a linear operator on V. Further, find the eigenvalues of T and an ordered basis β for V such that $[T]_\beta$ is a diagonal matrix.

(c) Test the linear operator $T: \mathbb{C}^2 \to \mathbb{C}^2$, $T(z, w) = (z + iw, iz + w)$ for diagonalizability and if diagonalizable, find a basis β for V such that $[T]_\beta$ is a diagonal matrix. (6,6,6)

4. (a) Let T be a linear operator on a finite dimensional vector space V and let W be a T-invariant subspace of V. State relationship between the characteristic polynomial of T_W and characteristic polynomial of T. Verify that relationship for the linear operator T: $\mathbb{R}^4 \rightarrow R^4$, $T(a, b, c, d) = (a + b + 2c - d, b + d, 2c - d, c + d)$ and the T-invariant subspace $W = \{(t, s, 0, 0): t, s \in \mathbb{R}\}$ of V.

(b) State Cayley-Hamilton theorem for an n-dimensional vector space V and use it to prove Cayley-Hamilton theorem for matrices.

(c) Let T be a linear operator on a vector space Land let $\lambda_1, \lambda_2, \ldots, \lambda_k$ be distinct eigenvalues of T. For each i = 1,2,..., k, let S_i be a finite linearly independent subset of the eigenspace E_{λ_i}. Then prove that $S = S_1 \cup S_2 \cup \cdots \cup S_k$ is a linearly independent subset of V. (6.5,6.5,6.5)

5\. (a) (i) Let $V = M_{2\times 2}(\mathbb{C})$ together with the Frobenius inner product given by

$\langle A, B\rangle = tr(B^*A)$ for all $A, B \in V$. Let

$A = \begin{bmatrix} 1 & 2+i \\ 3 & i \end{bmatrix}$ and $A = \begin{bmatrix} 1+i & 0 \\ i & -i \end{bmatrix}$. Compute

$\langle A, B\rangle$, $||A||$, $||B||$ and $||A+B||$. Then, verify both the Cauchy-Schwarz inequality and the triangle inequality.

(ii) Provide a reason why $\langle (a, b), (c, d)\rangle = ac - bd$ is not an inner product on $\mathbb{R}^2$.

(b) Apply the Gram-Schmidt process to a subset $S = \{\sin t, \cos t, 1, t\}$ of the inner product space $V = \text{span}(S)$ with the inner product

$\langle f, g\rangle = \int_0^{\pi} f(t)g(t)dt$ to obtain an orthogonal basis for V. Then, normalize the vectors in this basis to obtain an orthonormal basis β for V and compute the Fourier coefficients of the vector h(t) = 2t + 1 relative to β.

(c) Prove that an orthogonal subset of a finite-dimensional inner product space V can be extended to an orthonormal basis for V. Hence, or otherwise, prove that for any subspace W of a finite-dimensional inner product space V, $\dim(V) = \dim(W) + \dim(W^{\perp})$.

(4+2,6,6)

6. (a) Let T be a linear operator on a complex finite-dimensional inner product space V with an adjoint T*. Prove that

(i) T is self-adjoint if and only if $\langle Tx, x\rangle$ is real for all $x \in V$.

(ii) If $\langle Tx, x\rangle = 0$ for all $x \in V$, then $T = T_0$, the zero operator on V.

(b) (i) Show that the reflection operator on $\mathbb{R}^2$ about a line through the origin is an orthogonal operator.

(ii) Show that the pair of matrices $A = \begin{pmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

and $B = \begin{pmatrix} 1 & 0 & 0 \\ 0 & i & 0 \\ 0 & 0 & -i \end{pmatrix}$ are unitarily equivalent.

(c) (i) For the set of data S = {(–3,9), (–2,6), (0,2),

P.T.O.

(1,1)} use the least square approximation to find the line of best fit.

(ii) For $V = \mathbb{R}^2$ with the standard inner product and a linear operator $T: V \to V$, given by $T(a, b) = (2a + b, a - 3b)$ for all $a, b \in V$, evaluate $T^*(3,5)$.

(4+2.5,3+3.5,4+2.5)

[This question paper contains 8 printed pages.]

Your Roll No..............

Sr. No. of Question Paper : 3181 **H**

Unique Paper Code : 32357608

Name of the Paper : Mechanics

Name of the Course : **B.Sc. (H) Mathematics CBCS-LOCF**

Semester : VI

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.

2. **ALL** QUESTIONS are compulsory.

3. Attempt any **TWO PARTS** from each question.

4. Marks are indicated.

P.T.O.

1. (a) Find the moment of forces about the point A in the Figure 1. (6.5)

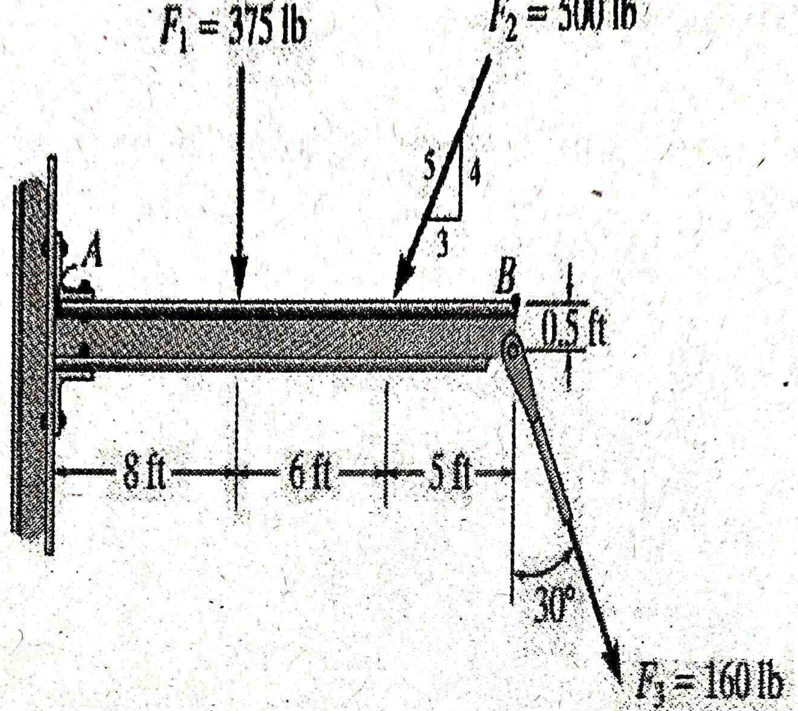


Figure 1

(b) Find the moment of the couple in the figure 2. (6.5)

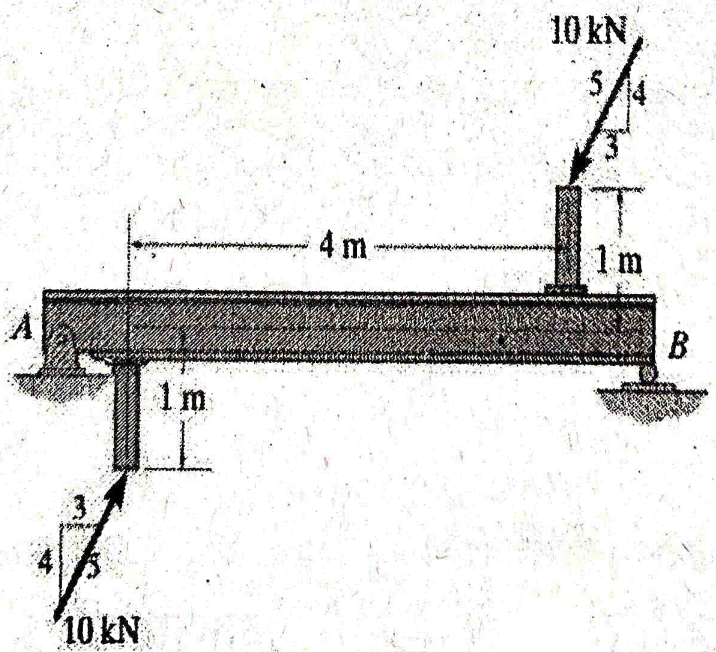


Figure 2

(c) Reduce the system of forces by a resultant' force and a couple at the point A in the Figure 3. (6.5)

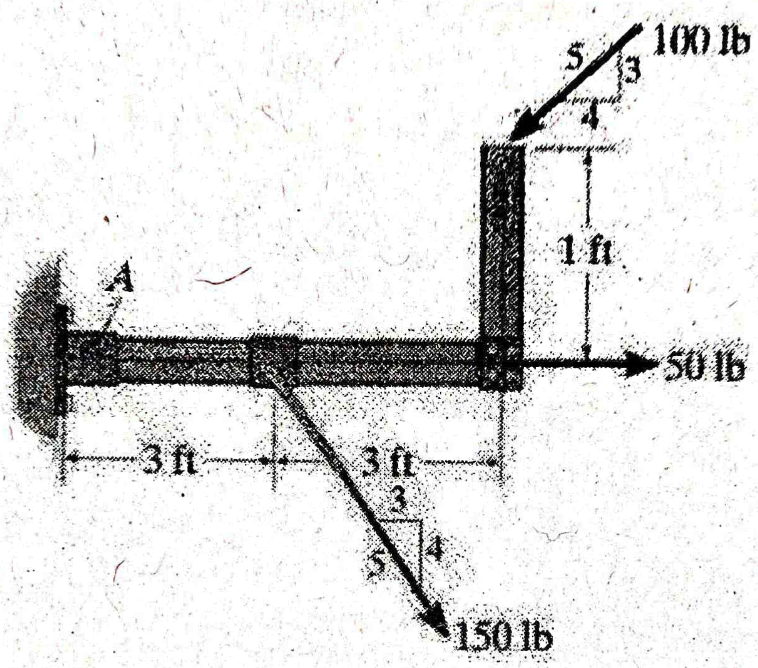


Figure 3

2. (a) A rod of 30 kg. is lying against a rough wall on a smooth horizontal surface. At the contact point on the smooth surface, a horizontal force is applied to stop the ladder from slipping. If the coefficient of static friction at wall is 0.2, then find the minimum horizontal force applied at the smooth surface. (6)

(b) Find the force P needed to cause impending motion of the block in the Figure 4. (6)

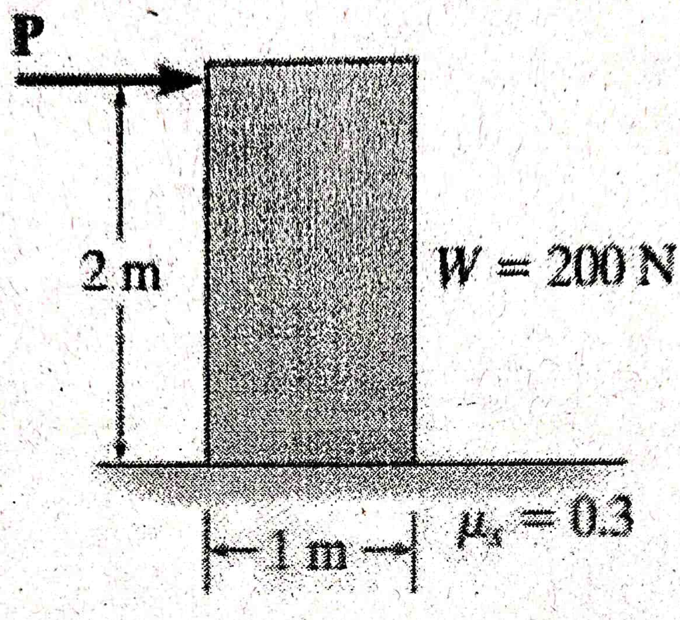


Figure 4

(c) Find the centroid of the shaded region in the Figure 5. (6)

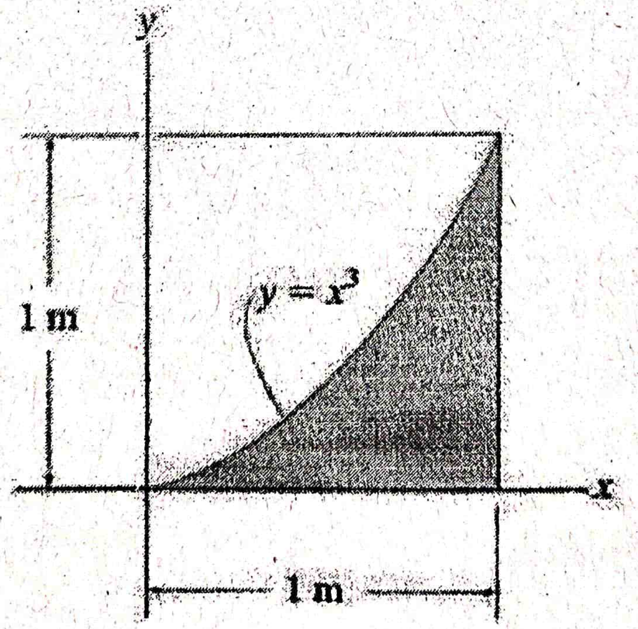


Figure 5

3. (a) Find the centroid of the solid in the Figure 6. (6.5)

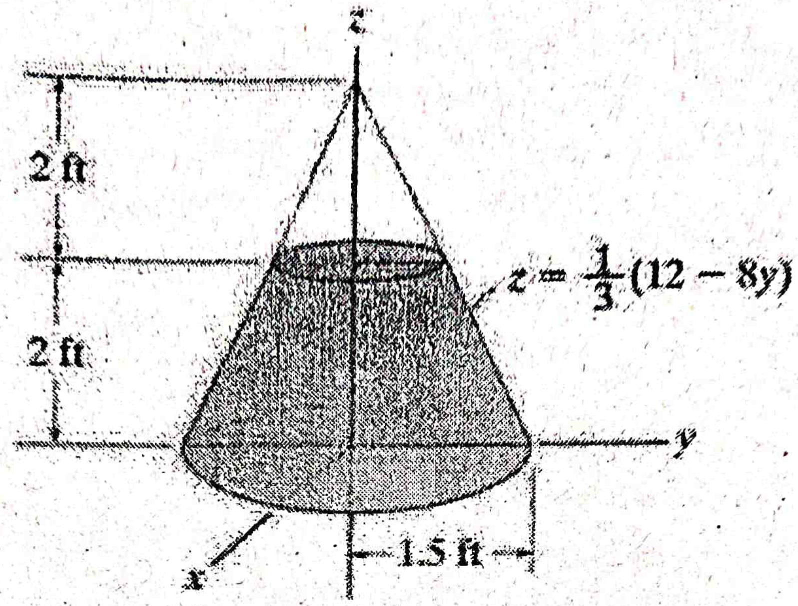


Figure 6

(b) Find the volume and the surface area of the solid made by revolving the shaded area 360° about the z-axis in the figure 7. (6.5)

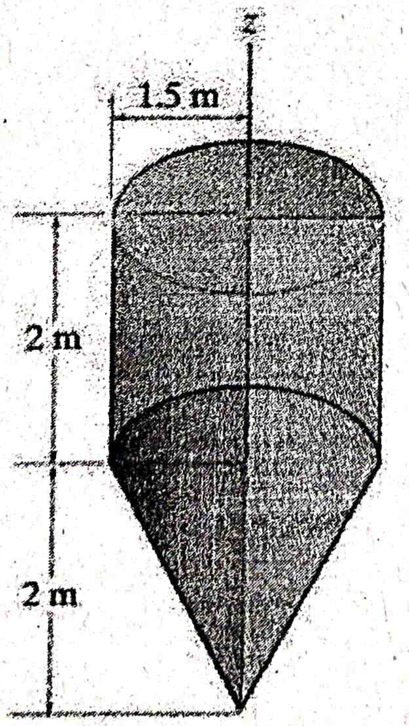


Figure 7

(c) Find the moment of inertia for the shaded region about the y-axis in the Figure 8. (6.5)

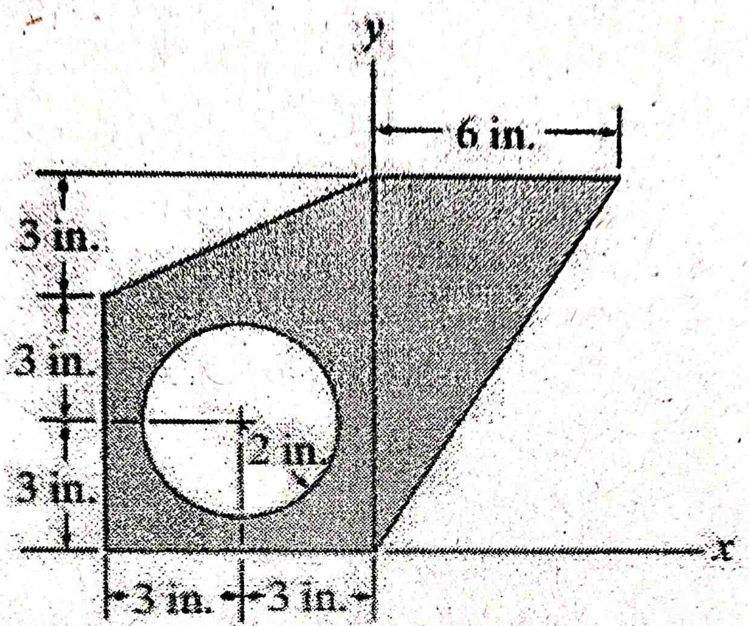


Figure 8

4. (a) Find the first moments of the area under the parabola $y^2 = 4x$ and $x = 1$, $y = 0$. (6)

(b) What are the second moments of area of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ for reference xy? (6)

(c) Show that the centroid of area under a semicircle in the first quadrant centred at (r, 0) is $\left(r, \frac{4r}{3\pi}\right)$, where r is the radius of circle. (6)

5. (a) A uniform cylinder of radius 1 m and of weight 100 lb rolls down a 30° incline without slipping. What is the speed of the centreline after it has moved 20 ft? (6.5)

(b) Determine whether the following force field is conservative or not

$F = (z \sin x + y)i + (2yz + 4x)j + (y^2 + 5 \cos x)k$ lb. (6.5)

(c) A chain of total length L is released from rest on a smooth horizontal support to fall down. Determine the velocity of the chain when the half of the length moves off the horizontal surface. Neglect the friction. (6.5)

6. (a) Show that derivative of a vector fixed in a moving reference is $\frac{d^2A}{dt^2} = (\omega \times A) \times A + \dot{\omega} \times A$. (6)

(b) A body is dropped from rest, determine the time required for it to acquire a velocity of 32 mt/Sec. (6)

(c) Define Translation and Rotation of rigid bodies. (6)

[This question paper contains 8 printed pages.]

Your Roll No..............

Sr. No. of Question Paper : 4083 **H**

Unique Paper Code : 2352011201

Name of the Paper : Linear Algebra

Name of the Course : **B.Sc. (H) Mathematics – DSC**

Semester : II

Duration : 3 Hours Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt **all** questions by selecting **two** parts from each question.
3. **All** questions carry equal marks.
4. Use of Calculator not allowed.

1. (a) If x and y are vectors in $\mathbb{R}^n$, then prove that $|x \cdot y| \le \|x\| \, \|y\|$. Also, verify the same for the vectors $x = [4,2,0,-3,-1]$ and $y = [1,4,-1,0,2]$ in $\mathbb{R}^5$.

(b) Using the Gauss – Jordan method, find the complete solution set for the following non-homogeneous system of linear equations :

$$2x_1 + 4x_2 - x_3 = 9$$

$$3x_1 - x_2 + 5x_3 = 5$$

$$8x_1 + 2x_2 + 9x_3 = 19$$

(c) Define the row space of a matrix. Also, determine whether the vector v = [7,1,18] is in the row space of the following matrix :

$$A = \begin{pmatrix} 3 & 6 & 2 \\ 2 & 10 & -4 \\ 2 & -1 & 4 \end{pmatrix}$$

If so, then express [7,1,18] as a linear combination of the rows of A.

2. (a) Define the rank of a matrix. Consider the matrix :

$$A = \begin{pmatrix} 1 & 2 & -1 \\ 2 & -1 & 3 \\ 5 & -4 & 3 \end{pmatrix}$$

Using rank of A, determine whether the homogeneous system AX = 0 has a non-trivial solution or not.

(b) Consider the matrix :

$$A = \begin{pmatrix} 4 & 0 & 4 \\ 0 & 4 & 4 \\ 4 & 4 & 8 \end{pmatrix}.$$

(i) Find the eigenvalues and the fundamental eigenvectors of A.

(ii) Is A diagonalizable? Justify your answer.

(c) Find the quadratic equation of the form $y = ax^2 + bx + c$ that passes through the points $(-2,20)$, $(1,5)$, and $(3,25)$.

3. (a) (i) Let $\mathcal{C}$ be a family of subspaces of a vector space V and let W denote the intersection of subspaces in $\mathcal{C}$. Prove that W is a subspace of V.

(ii) Let F be any field. Prove that $W = \{(a_1, a_2, \ldots, a_n) \in F^n \mid a_1 + a_2 + \cdots a_n = 0\}$ is a subspace of F^n.

(b) Let W_1 and W_2 be subspaces of a vector space V. Prove that V is the direct sum of W_1 and W_2 if and only if each vector in V can be uniquely written as $x_1 + x_2$, where $x_1 \in W_1$ and $x_2 \in W_2$.

(c) (i) Show that if S_1 and S_2 are arbitrary subsets of a vector space V_1 then

$$\text{span}(S_1 \cap S_2) \subseteq \text{span}(S_1) \cap \text{span}(S_2).$$

Give an example in which $\text{span}(S_1 \cap S_2)$ and $\text{span}(S_1) \cap \text{span}(S_2)$ are unequal.

(ii) Does the polynomial $-x^3 + 2x^2 + 3x + 5$ belong to span (S), where

$$S = \{x^3 + x + 1, x^3 - 2x^2 + 1, x^2 + x + 1).$$

4. (a) Let S be a linearly independent subset of a vector space V, and let v be a vector in V that is not in S. Prove that $S \cup \{v\}$ is linearly dependent if and only if $v \in \text{span}(S)$.

(b) Let $V = M_{2\times2}(F)$, $W_1 = \left\{\begin{pmatrix} a & b \\ c & a \end{pmatrix} \in V \mid a, b, c \in F\right\}$ and

$$W_2 = \left\{\begin{pmatrix} 0 & a \\ -a & b \end{pmatrix} \in V \mid a, b \in F\right\}.$$

Find a basis for the subspaces W_1, W_2 and $W_1 \cap W_2$. Also, find the dimension of each of them.

(c) Let W be a subspace of a finite dimensional vector space V. Prove that W is finite dimensional and dim(W) ≤ dim(V). Further, if dim(W) = dim(V), then show that V = W.

5. (a) Let V and W be two finite dimensional vector spaces and let T: V → W be a linear transformation. Then prove that

$$\text{nullity}(T) + \text{rank}(T) = \dim(V)$$

Also, prove that if dim(V) < dim(W), then T cannot be onto.

(b) Let $T: \mathbb{R}^3 \to \mathbb{R}^2$ be defined by $T(a_1, a_2, a_3) = (-2a_1 + 3a_3, a_1 + 2a_2 - a_3)$ with $\beta = \{(1,-3,2), (-4,13,-3), (2,-3,20)\}$ and $\gamma = \{(-2,-1), (5,3)\}$. Compute $[T]_\beta^\gamma$. Is T one to one?

(c) Let $L: \mathbb{R}^3 \to \mathbb{R}^3$ be a linear transformation defined by :

$$L\begin{pmatrix} u_1 \\ u_2 \\ u_3 \end{pmatrix} = \begin{pmatrix} u_1 + u_3 \\ u_1 + u_2 \\ u_2 - u_3 \end{pmatrix}$$

Find bases for null space N(T) and range space R(T). Also, verify the dimension theorem.

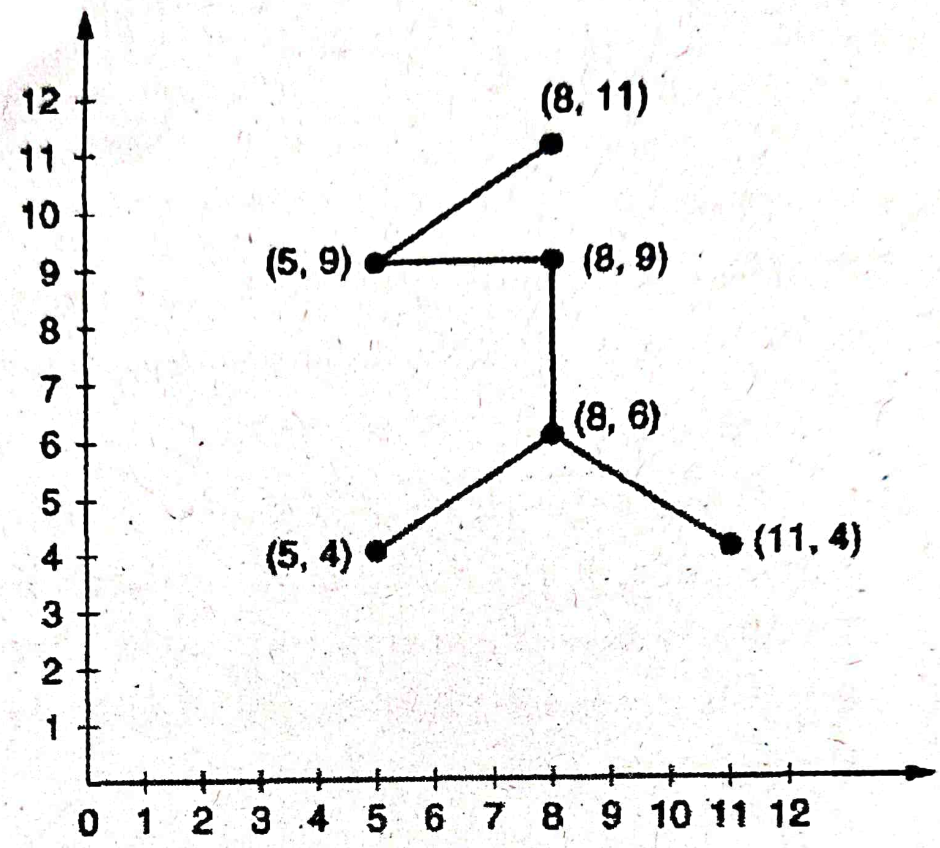
6. (a) Let V and W be finite dimensional vector spaces with ordered basis β and γ respectively. Let $T: V \to W$ be a linear transformation. Then prove that T is invertible if and only if $[T]_\beta^\gamma$ is invertible. Furthermore, $[T^{-1}]_\gamma^\beta = ([T]_\beta^\gamma)^{-1}$.

(b) Let V and W be finite dimensional vector spaces and let $T: V \to W$ be an isomorphism. Let V_0 be a subspace of V.

(i) Prove that $T(V_0)$ is a subspace of W.

(ii) Prove that $\dim(V_0) = \dim(T(V_0))$.

(c) For the adjoining graphic, use homogenous co-ordinates to find the new vertices after performing scaling about (8,4) with scale factors of 2 in the x-direction and 1/3 the y-direction.



Also, sketch the final figure that would result from this movement.

[This question paper contains 4 printed pages.]

Your Roll No.............

Sr. No. of Question Paper : 4121 **H**

Unique Paper Code : 2352011202

Name of the Paper : CALCULUS – DSC 5

Name of the Course : **B.Sc. (H) Mathematics UGCF-2022**

Semester : II

Duration : 3 Hours Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt all questions by selecting **three** parts from each question.
3. **All** questions carry equal marks.
4. Use of Calculator is not allowed.

1. (a) If f: A → $\mathbb{R}$ and if c is a cluster point of A then prove that f can have only one limit at c. (5)

(b) Use $\in - \delta$ definition of limit to establish the following limit : (5)

$$\lim_{x \to -1} \frac{x+5}{2x+3} = 4.$$

P.T.O.

(c) Determine whether the following limit exists in $\mathbb{R}$

$$\lim_{x \to 0} sgn \sin\left(\frac{1}{x}\right) \quad (5)$$

(d) Let $A \subseteq \mathbb{R}$, let f, g, h: $A \to \mathbb{R}$, and let $c \in \mathbb{R}$ be cluster point of A. If $f(x) \leq g(x) \leq h(x)$ for all $x \in A$, $x \neq c$ and if $\lim_{x \to c} f = L = \lim_{x \to c} h$, then show that $\lim_{x \to c} g = L$. (5)

2. (a) State and prove sequential criterion for continuity. (5)

(b) Let $f : \mathbb{R} \to \mathbb{R}$ be defined as

$$f(x) = \begin{cases} 1 & \text{if } x \text{ is rational} \\ 0 & \text{if } x \text{ is irrational} \end{cases}$$

Show that f is discontinuous at every x in $\mathbb{R}$. (5)

(c) Let f and g be continuous real-valued function on (a, b) such that f(r) = g(r) for each rational number r in (a, b) then prove that f(x) = g(x) for all $x \in (a, b)$. (5)

(d) State Intermediate Value Theorem. Show that $x = \cos x$ for some x in $\left(0, \frac{\pi}{2}\right)$. (5)

3. (a) Prove that every continuous function defined on a closed interval is bounded therein. (5)

(b) If f is uniformly continuous on a set S and (s_n) is a Cauchy sequence in S, then prove that $(f(s_n))$ is a Cauchy sequence. (5)

(c) Show that the function $f(x) = \frac{1}{x^2}$ is not uniformly continuous on (0,1) but it is uniformly continuous on $[a, \infty)$ where $a > 0$. (5)

(d) Let $f(x) = x^2 \sin\frac{1}{x}$ for $x \neq 0$ and $f(0) = 0$. Show that f is differentiable on $\mathbb{R}$ and also show that f′ is not continuous at $x \neq 0$. (5)

4. (a) Let f be defined on an open interval containing x_0. If f assumes its maximum or minimum at x_0 and if f is differentiable at x_0 then show that $f'(x_0) = 0$. (5)

(b) State Intermediate Value Theorem for derivatives. Suppose f is differentiable on E and $f(0) = 0$, $f(1) = 1$, $f(2) = 1$.

(i) Show that $f'(x) = \frac{1}{3}$ for some $x \in (0,2)$.

(ii) Show that $f'(x) = \frac{2}{5}$ for some $x \in (0,2)$.

(5)

(c) Show that $ex \leq e^x$ for all $x \in \mathbb{R}$. (5)

(d) Suppose f is differentiable on $\mathbb{R}$, $1 \le f'(x) \le 2$ for $x \in \mathbb{R}$, and $f(0) = 0$. Prove $x \le f(x) \le 2x$ for all $x \ge 0$. (5)

5. (a) Let f be differentiable function on an open interval (a, b). Then show that f is strictly increasing on (a, b) if $f'(x) > 0$. (5)

(b) If $y = \sin^{-1} x$, prove that (5)

$$(1 - x^2)y_{n+2} - (2n + 1)x\,y_{n+1} - n^2y_n = 0.$$

(c) If $y = \left[x + \sqrt{1+x^2}\right]^m$, find $y_n(0)$. (5)

(d) State Taylor's theorem. Find Taylor series expansion of cos x. (5)

6. (a) Find all values of k and l such that

$$\lim_{x \to 0} \frac{k + \cos lx}{x^2} = -4.$$ (5)

(b) Determine the position and nature of the double points on the curve (5)

$$y^2 = (x - 2)^2(x - 1).$$

(c) Sketch a graph of the rational function showing the horizontal, vertical and oblique asymptote (if any) of $y = x^2 - \frac{1}{x}$. (5)

(d) Sketch the curve in polar coordinates of $r = \cos 2\theta$. (5)

[This question paper contains 8 printed pages.]

Your Roll No..............

Sr. No. of Question Paper : 4159 **H**

Unique Paper Code : 2352011203

Name of the Paper : Ordinary Differential Equations

Name of the Course : **B.Sc. (Hons.) Mathematics – DSC**

Semester : II

Duration : 3 Hours Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt any **two** parts of each question.
3. Each part carries **7.5** marks.
4. Use of non-programmable Scientific Calculator is allowed.

1. (a) Solve

$$\{y^2(x + 1) + y\}dx + (2xy + 1)dy = 0$$

P.T.O.

(b) Solve the initial value problem

$$x\frac{dy}{dx}\ y = y^2 \log x,\ y(1) = 2$$

(c) (i) Solve

$$(y \sec^2 x + \sec x \tan x)dx + (\tan x + 2y)dy = 0$$

(ii) Solve by reducing the order $y'' = (x + y')^2$

2. (a) Suppose that a mineral body formed in an ancient cataclysm originally contained the uranium isotope ^{238}U, (which has a half-life of 4.51×10^9 years) but no lead, the end product of the radioactive decay of ^{238}U. If today the ratio of ^{238}U atoms to lead atoms in the mineral body is 0.9, when did the cataclysm occur?

(b) Upon the birth of their first child, a couple deposited Rs. 10,000 in an account that pays 8% interest compounded continuously. The interest payment is allowed to accumulate. In how many years will the amount double? How much will the account contain on the child's 18^{th} birthday?

(c) A roast initially at 50°F, is placed in a 375°F oven at 5 pm. After 75 minutes, it is found that the temperature of the roast is 125°F. When will the roast be 150°F?

3. (a) Show that the solutions e^x, e^{-x}, e^{-2x} of the third order differential equation

$$\frac{d^3y}{dx^3}+2\frac{d^2y}{dx^2}-\frac{dy}{dx}-2y=0$$

are linearly independent. Find the particular solution satisfying the given initial condition.

$$y(0)=1,\ y'(0)=2,\ y''(0)=0$$

(b) Solve the differential equation using the method of Variation of Parameters

$$\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y = 2e^{2x}$$

(c) Find the general solution of the differential equation using the method of Undetermined Coefficients.

$$\frac{d^2y}{dx^2} + 9y = 2\cos 3x$$

4. (a) Use the operator method to find the general solution of the following linear system

$$\frac{dx}{dt} + \frac{dy}{dt} - 2x - 4y = e^t$$

$$\frac{dx}{dt} + \frac{dy}{dt} - y = e^{4t}$$

(b) Find the general solution of the differential equation. Assume $x > 0$.

$$x^2 \frac{d^2y}{dx^2} + x\frac{dy}{dx} - y = 72x^5$$

(c) A body with mass $m = \frac{1}{5}$ kg is attached to the end of a spring that is stretched 4m by a force of 20N. It is set in motion with initial position $x_0 = 1$m and initial velocity $v_0 = -5$m/s. Find the position function of the body as well as the amplitude, frequency and period of oscillation.

5. (a) Develop a model with two differential equations describing a predator-prey interaction.

(b) Define equilibrium solution of differential equation. A model of a three species interaction with two predators that compete for a single prey food-source is

$$\frac{dX}{dt} = a_1X - b_1XY - c_1XZ, \frac{dY}{dt} = a_2XY - b_2Y, \frac{dZ}{dt} = a_3XZ - b_3Z$$

where a_i, b_i, c_i for i = 1,2,3 are all positive constants. Here X(t) is the prey density and Y(t), Z(t) are the two predator densities. Find all possible equilibrium populations.

(c) Suppose a population can be modeled using the differential equation

$$\frac{dX}{dt} = 0.2X - 0.001X^2$$

with an initial population size of x_0 = 100 and a time step of 1 month. Find the predicted population after 2 months.

6. (a) The Earth's atmospheric pressure p is often modelled by assuming that $\frac{dp}{dx}$ (the rate at which pressure p changes with altitude h above sea level) is proportional to p. Suppose that the pressure at sea level is 1013 millibars and that the pressure at an altitude of 20 km is 50 millibars. Use an exponential decay model

$$\frac{dp}{dx} = kp$$

to describe the system, and then by solving the equation, find an expression for p in terms of h. Determine k and the constant of integration from the initial conditions. What is the atmospheric pressure at an altitude of 50 km?

(b) Discuss Phase Plane Analysis of predator-prey model.

(c) A large tank contains 100 litres of salt water. Initially s_0 kg of salt is dissolved. Salt water flows into the tank at the rate of 10 litres per minute, and the concentration $c_{in}(t)$ (kg of salt/litre) of this incoming water-salt mixture varies with time. We assume that the solution in the tank is thoroughly mixed and that the salt solution flows out at the same rate at which it flows in: that is, the volume of water-salt mixture in the tank remains constant. Find a differential equation for the amount of salt in the tank at any time t.

[This question paper contains 4 printed pages.]

Your Roll No...............

Sr. No. of Question Paper : 5037 **H**

Unique Paper Code : 2352201202

Name of the Paper : DSC : Analytic Geometry

Name of the Course : **Bachelor of Arts**

Semester : II

Duration : 3 Hours Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. **All** questions are compulsory.
3. Attempt any **two** parts from each section.
4. **All** questions carry equal marks.

1. (a) Show that the equation of the parabola with axis y=0 and passing through (3, 2), and (2, –3) is

$$y^2 = -5\left(x - \frac{19}{5}\right).$$ Also sketch the graph.

P.T.O.

(b) Identify and sketch the curve

$9x^2 + 4y^2 - 18x + 24y + 9 = 0.$

(c) Describe the graph of the equation

$x^2 - 4y^2 + 2x + 8y - 7 = 0.$

2. (a) Find the equation of the parabola that has its vertex at (1, 2) and focus at (4, 2). Also, state the reflection property of the parabola.

(b) Identify and sketch the curve $3x^2 + 2xy + 3y^2 = 19$.

(c) A box is dragged along the floor by a rope that applies a force of 50 lb at an angle of 60° with the floor. How much work is done in moving the box 15 ft?

3. (a) Express the vector v as the sum of a vector parallel to $\vec{b}$ and a vector orthogonal to $\vec{b}$ where $\vec{v} = -2\hat{i} + \hat{j} + 6\hat{k}$, $\vec{b} = -2\hat{j} + \hat{k}$.

(b) Find two unit vectors that are orthogonal to both $\vec{u} = -7\hat{i} + 3\hat{j} + \hat{k}$ and $\vec{v} = 2\hat{i} + 4\hat{k}$.

(c) Use a scalar triple product to determine whether the vectors $\vec{u} = \hat{i} - 2\hat{j} + \hat{k}$, $\vec{v} = 3\hat{i} - 2\hat{k}$ and $\vec{w} = 5\hat{i} - 4\hat{j}$ lie in the same plane.

4. (a) Find the parametric equation of the line that passes through origin and is parallel to the line L: $x = t$, $y = -1 + t$, $z = 2$.

(b) Find the direction cosines of two lines that are connected by the relations

$l + m - n = 0$, $mn + 6ln - 12lm = 0$.

(c) Find the equation of the plane through the points P(–2,1,4) and Q(1,0,3) that is perpendicular to the plane $4x - y + 3z = 2$.

5. (a) Find the centre and the radius of the circle

$x + 2y + 2z = 15$, $x^2 + y^2 + z^2 - 2y - 4z = 11$.

(b) Prove that the tangent planes to the cone

$$x^2 - y^2 + 2z^2 - 5xy - 3yz + 4zx = 0$$

are perpendicular to the generators of the cone

$$17x^2 + 8y^2 + 29z^2 - 16xy + 28yz - 46zx = 0.$$

(c) Find the equation of the right circular cylinder of radius 2 whose axis is the line $\frac{x-1}{2}=\frac{y-2}{2}=\frac{z-2}{2}$.

6. (a) Show that the plane $2x - 2y + z + 12 = 0$ touches the sphere $x^2 + y^2 + z^2 - 2x - 4y + 2z = 3$ and find the point of contact.

(b) Show that the equation of the right circular cone with vertex (2, 3, 1), axis parallel to the line $-x=\frac{y}{2}=z$ one of its generators having direction cosines proportional to (1, –1, 1) is

$x^2 - 8y^2 + z^2 + 12xy - 12yz + 6zx - 46x + 36y + 22z - 19 = 0$.

(c) Find the equation of a cylinder whose generating lines have the direction cosines (l, m, n) and which passes through the circle $x^2 + z^2 = a^2$, $y = 0$.

[This question paper contains 4 printed pages.]

Your Roll No..............

Sr. No. of Question Paper : 4102 **H**

Unique Paper Code : 2352012402

Name of the Paper : Multivariate Calculus

Name of the Course : B.A./B.Sc. (H)

Semester : IV (DSC)

Duration : 3 Hours Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt **all** questions by selecting **two** parts from each question.
3. **All** questions carry equal marks.
4. Use of Calculator not allowed.

1. (a) Let f be the function defined by

$$f(x,y)=\begin{cases}\dfrac{xy^3}{x^2+y^6}, & (x,y)\neq(0,0)\\ 0, & (x,y)=(0,0)\end{cases}$$

Is f continuous at (0,0)? Explain.

P.T.O.

(b) A cylindrical tank is 4 ft high and has an outer diameter of 2 ft. The walls of tank are 0.2 inches thick. Approximate the volume of interior of tank assuming the tank has a top and a bottom that are both 0.2 inches thick.

(c) Find an equation for each horizontal tangent plane to the surface

$$z = 5 - x^2 - y^2 + 4y.$$

2. (a) Let $f(x, y, z) = ye^{x+z} + ze^{y-x}$. At the point (2,2,–2) find the unit vector pointing in the direction of most rapid increase of f. And what is the value of most rapid increase of f?

(b) Find the absolute extrema of the function $f(x, y) = 2 \sin x + 5 \cos y$ on the set S where S is the rectangular region with vertices (0,0), (2,0), (2,5) and (0,5).

(c) Find the point on the plane $2x + y + z = 1$ that is nearest to the origin.

3. (a) (i) Sketch the region of integration and compute the double integral

$$\int_0^{\pi/2} \int_0^{\sin x} e^x \cos x \, dydx .$$

(ii) Evaluate $\iint_D (2y - x)dA$ where D is the region bounded by $y = x^2$, $y = 2x$.

(b) Evaluate the area bounded by $r = 2\cos\theta$.

(c) Evaluate the given integral by converting to polar coordinates

$$I = \int_0^2 \int_y^{\sqrt{8-y^2}} \frac{1}{\sqrt{1+x^2+y^2}} dydx .$$

4. (a) (i) Set up a triple integral to find volume of solid bounded above by paraboloid $z = 6 - x^2 - y^2$ and below by $z = 2x^2 + y^2$.

(ii) Set up a double integral to find volume of solid region bounded by ellipsoid

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1 .$$

(b) Use cylindrical co-ordinates to compute integral

$\iiint_D z\left(x^2+y^2\right)^{-1/2} dxdydz$ where D is the solid bounded above by plane $z = 2$ and bounded below by surface $2z = x^2 + y^2$.

(c) Find volume of solid D where D is the intersection of solid sphere $x^2 + y^2 + z^2 \leq 9$ and solid cylinder $x^2 + y^2 \leq 1$.

5. (a) Evaluate $\oint_C (x^2y\,dx - xy\,dy)$, where C is the path that begins at (0,0), goes to (1,1) along the parabola $y = x^2$, and then return to (0,0) along the line $y = x$.

(b) Prove or disprove that the line integrals are path independent.

(c) Evaluate the line integral

$$\oint_C [(2x - x^2y)e^{-xy} + \tan^{-1} y]\boldsymbol{i} + [\frac{x}{y^2+1} - x^3e^{-xy}]\boldsymbol{j}.dR,$$

where C, the curve with parametric equations $x = t^2 \cos \pi t$, $y = e^{-t} \sin \pi t$, $0 \le t \le 1$.

6. (a) Find the area enclosed by the semicircle $y = \sqrt{4 - x^2}$ using the line integral.

(b) Find the mass of the homogenous lamina of density $\delta = x$ that has the shape of the surface S given by $z = 4 - x - 2y$ with $z \ge 0$, $x \ge 0$ and $y \ge 0$.

(c) Let $F = 2xi - 3yj + 5zk$, and let S be the hemisphere $z = \sqrt{9 - x^2 - y^2}$ together with the disk $x^2 + y^2 \le 9$ in the xy–plane. Verify the divergence theorem.

(2000)

[This question paper contains 12 printed pages.]

Your Roll No..............

Sr. No. of Question Paper : 4227 **H**

Unique Paper Code : 2353012005

Name of the Paper : Mathematical Modeling

Name of the Course : **B.Sc. (H) – DSE**

Semester : IV

Duration : 3 Hours Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.

2. Each question contains three parts. Attempt all questions by selecting any two parts from each question.

3. Parts of questions to be attempted together.

4. **All** questions carry equal marks

5. Use of non-programmable scientific calculators is allowed.

P.T.O.

1. (a) (i) Define Mathematical modeling. Explain the modeling cycle process. A ball and bat together cost \$1.10. The bat costs \$1.00 more than the ball. Model this problem by using mathematical symbols and determine how much does the ball cost.

(ii) Compute the dimensions of a, D, λ_1 and λ_2 from the following differential equation, assuming that it is a dimensionally consistent equation.

$$\frac{\partial u}{\partial t} + au\frac{\partial u}{\partial x} = D\frac{\partial^2 u}{\partial x^2} + \lambda_1\sqrt{u} - \lambda_2 u^2$$

(b) In a particular epidemic model, where the infected individuals eventually recover, the population dynamics are governed by the following system of differential equations :

$$\frac{dS}{dt} = -\frac{\beta SI}{N}, \frac{dI}{dt} = \frac{\beta SI}{N} - \gamma I.$$

Given the parameter values $\beta = 2$ and $\gamma = 0.4$ and assuming that the total population N is 1000, in which initially there is only one infected individual but there are 999 susceptible individuals within the population, answer the following :

(i) Using the given parameters, calculate the basic reproduction number (r_0).

(ii) What is the peak number of infected individuals at any given time during the epidemic?

(c) Consider the following susceptible-infected-recovered (SIR) model :

$$\frac{dS}{dt} = -\beta\sqrt{S}\sqrt{I}, \frac{dI}{dt} = \beta\sqrt{S}\sqrt{I} - \gamma\sqrt{I}, \frac{dR}{dt} = \gamma\sqrt{I},$$

where β and γ are constant parameters and initial subpopulations are given as $S(0) = S_0 > 0$, $I(0) = I_0 > 0$, $R(0) = 0$. By taking the first two equations of the model for analysis purposes, and assuming the transformation $u = \sqrt{S}$, $v = \sqrt{I}$, show that

$$v(t) = \left[\left(\sqrt{S_0} - \sqrt{S^*}\right)^2 + I_0\right]^{\frac{1}{2}} . \sin\left(\frac{\beta}{2}t + \phi_1\right),$$

where $\phi_1 = \tan^{-1}\left(\frac{\sqrt{I_0}}{\sqrt{S_0}-\sqrt{S^*}}\right)$ and $S^* = \frac{\gamma}{\beta}$.

2. (a) Consider an epidemic model, where the infected individuals eventually recover and the dynamics of the population are described by ≥ differential equations :

$$\frac{dS}{dt} = -\beta\sqrt{S}\sqrt{I}, \frac{dI}{dt} = \beta\sqrt{S}\sqrt{I} - \gamma\sqrt{I}.$$

Using the parameter values $\beta = 0.02$ and $\gamma = 0.4$ and assuming initially there is only one infected individual but there are only 500 susceptible individuals within a population,

(i) Calculate the basic reproduction number (r_0) using the given parameters.

(ii) How many individuals remain susceptible and never get infected throughout the epidemic?

(iii) What is the maximum number of individuals infected at any point in time during the epidemic?

(b) Construct a Susceptible-Exposed-Infectious-Recovered (SEIR) model using a system of first-order differential equations to describe the spread of disease. Divide the total population N(t) at any time t into subpopulations according to disease status. Assuming a constant influx of individuals into the susceptible population and a natural death rate affecting each subpopulation, as well as the exposure rate of susceptible individuals and recovery rate of infectious people, obtain the exact solution for the total population N(t), and provide the discretized form of the model equations using the Nonstandard Finite Difference (NSFD) scheme.

(c) Consider the following susceptible-infected-recovered (SIR) model :

$$\frac{dS}{dt} = -\beta S\frac{I}{N}, \frac{dI}{dt} = \beta S\frac{I}{N} - \gamma I, \frac{dR}{dt} = \gamma I,$$

where β and γ are constant parameters and initial subpopulations are :

$S(0) = S_0 > 0,\ I(0) = I_0 > 0,\ R(0) = R_0 \geq 0.$

(i) Find the condition for epidemic occurrence by finding the basic reproduction number (r_0).

(ii) If the first integral expression in the (S, I) plane is given as $S(t) + I(t) = I_0 + S_0 + S^*\ln\left(\frac{S(t)}{S_0}\right)$, where $S^* = \frac{\gamma}{\beta}N$, then find the expressions for the maximum infective number (I_{max}), and the number of susceptibles who do not succumb to the epidemic (S_∞).

3. (a) Explain and analyze the Predator-Prey system (Lotka-Volterra model) by identifying and discussing all isolated critical points.

(b) Find all critical points of the system of differential equations

$$\begin{cases} \frac{dx}{dt} = 1 - xy \\ \frac{dy}{dt} = x - y^3 \end{cases}$$

and determine (if possible) whether they are stable or unstable.

(c) Consider a damped nonlinear spring-mass system. Let m denote the mass of an object attached to a spring and let x(t) denote the displacement of the mass at time t from its equilibrium position. Assume that the mass on the spring is connected to a dashpot, exerting a force of resistance proportional to velocity, $y = x' = \frac{dx}{dt}$ of the mass. Let the force exerted by the spring on the mass be given by by $F(x) = m\frac{dy}{dt} = mx''$. The equation of motion of the mass is given by mx″ = –cx′ – kx +

βx^3, where c, m, k, $\beta > 0$. If $c = 2$, $\beta = \frac{5}{4}$, $k = 5$, and $m = 1$, then write the corresponding system of first-order differential equations and discuss the stability of all critical points of the system.

4. (a) Consider the system of differential equations :

$$\begin{cases} \frac{dx}{dt} = x + \epsilon y \\ \frac{dy}{dt} = x - y \end{cases}.$$

Determine the conditions on e for which the critical point of the system may be a saddle point and a center in the phase plane and identify the points where this change occurs.

(b) Determine the type and stability of the critical point (x*, y*) for the following systems :

(i) $\frac{dx}{dt} = 33 - 10x - 3y + x^2, \frac{dy}{dt} = -18 + 6x + 2y - xy;$

Critical point: (x*,y*) = (4,3).

(ii) $\frac{dx}{dt} = 3x - x^2 - xy, \frac{dy}{dt} = y + y^2 - 3xy;$

Critical point: $(x^*, y^*) = (1,2)$.

(c) Consider the following system of differential equations :

$$\begin{cases} \frac{dx}{dt} = 60x - 3x^2 - 4xy \\ \frac{dy}{dt} = 42y - 3y^2 - 2xy \end{cases}$$

Show that the linearization of the system at (20,0) is

$$\begin{cases} u' = -60u - 80v \\ v' = 2v \end{cases} \quad \text{where } u' = \frac{du}{dt} \text{ and } v' = \frac{dv}{dt}.$$

Find the eigenvalues and corresponding eigenvectors of the coefficient matrix of the linear system. Hence, confirm the nature of the critical point (20,0).

5. (a) Using Monte Carlo simulation write an algorithm to calculate an approximate area trapped between the curves $y = x$ and $y^2 = 2x$.

(b) Explain the Linear Congruence Method of generating random numbers. List the drawbacks of this method. Find 10 random numbers using a = 5, b = 1, c = 8 and 7 as the seed number.

(c) Using Monte Carlo simulation, write an algorithm to calculate that part of the volume of an ellipsoid $\frac{x^2}{2}+\frac{y^2}{4}+\frac{z^2}{8} \leq 16$ that lies in the first octant, $x > 0$, $y > 0$, $z > 0$.

6. (a) Use the Simplex Method to solve the following problem

$$\text{Max. } z = 3x + 2y$$

s. to
$$-x + 2y \leq 4,$$
$$5x + 2y \leq 14,$$
$$x - y \leq 3,$$
$$x, y \geq 0.$$

(b) Consider the following Linear Programming problem :

Max. $z = x + y$

s. to $x + y \leq 6,$

$3x - y \leq 9,$

$x, y \geq 0.$

Using algebraic methods find the possible number of points of intersection in the xy- plane. Are all the points feasible? If not, then how many are feasible and how many are infeasible. List the feasible extreme points along with the value of the objective function

(c) Solve the following Linear Programming problem graphically.

Max. $z = 20x + 30y$

s. to $3x + 2y \leq 80,$

$2x + 4y \leq 120,$

$x, y \geq 0.$

Perform a sensitivity analysis to find the range of values for the coefficient of x in the objective function for which the current extreme point remains optimal.

[This question paper contains 8 printed pages.]

Your Roll No..............

Sr. No. of Question Paper : 3071 **H**

Unique Paper Code : 32357614

Name of the Paper : DSE-3 MATHEMATICAL FINANCE

Name of the Course : **B.Sc. (Hons) Mathematics CBCS (LOCF)**

Semester : VI

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.

2. Attempt any **two** parts from each question.

3. **All** questions are compulsory and carry equal marks.

4. Use of Scientific calculator, Basic calculator and Normal distribution tables all are allowed.

P.T.O.

1. (a) Define Convexity of a bond and find the relation between the Convexity, Duration and Bond price. How does convexity measure sensitivity of the portfolio?

(b) Consider the three bonds having payments as shown in the table below. They are traded to procure a 12% yield with continuous compounding.

End of year payments	Bond A	Bond B	Bond C
Year 1	1000	500	0
Year 2	1000	500	0
Year 3	1000+10000	500+10000	0+10000

Determine the price and duration of each bond.

(c) Explain Forward Rates. Derive the following relation :

$$R_F = R_2 + (R_2 - R_1)\frac{T_1}{T_2 - T_1}$$

where R_F is the forward interest rate for the period between T_1 and T_2. R_1 and R_2 are the zero rates for maturities T_1 and T_2 respectively. What happens when $R_{[illegible]} > R_{[illegible]}$?

2. (a) (i) An investor sells a European call option with strike price of K and maturity T and buys a put with the same strike price and maturity. Describe the investor's position.

(ii) An investor sells a European call on a share for ₹6, the stock price is ₹45 and the strike price is ₹52. Under what circumstances will the seller of the option make a profit? Under what circumstances will the option be exercised?

(b) Write a short note on European put options. Explain the payoffs in different types of put option positions with the help of diagrams.

(c) Explain Hedging. A United States company expects to pay 1 million Euros in 3 months. Explain how the exchange rate risk can be hedged using

(i) A Forward Contract

(ii) An Option.

3. (a) Name the six factors that affect stock option prices. Explain any three of them.

(b) Derive the put-call parity for European options on a non-dividend-paying stock. Use put-call parity to derive the relationship between the gamma of a European call and the gamma of a European put on a non-dividend-paying stock.

(c) Find lower bound and upper bound for the price of a 1-month European put option on a non-dividend-paying stock when the stock price is ₹30, the strike price is ₹34, and the risk-free interest rate is 6% per annum? Justify your answer with no arbitrage arguments. ($e^{-0.005} = 0.9950$).

4. (a) A stock price is currently ₹40. It is known that at the end of one month it will be either ₹42 or ₹38. The risk-free interest rate is 6% per annum with continuous compounding. Consider a portfolio consisting of one short call and A shares of the stock. What is the value of A which makes the

portfolio riskless? Using no-arbitrage arguments, find the price of a one-month European call option with a strike price of ₹39? (You can use exponential value: $e^{0.005} = 1.005$).

(b) A stock price is currently ₹50. Over each of the next two three-month periods it is expected to go up by 6% or down by 5%. The risk-free interest rate is 5% per annum with continuous compounding. What is the value of a six-month European call option with a strike price of ₹51? (You can use exponential value: $e^{-0.0125} = 0.9876$).

(c) Consider a two-period binomial model with current stock price S_0 = ₹100, the up factor $u = 1.2$, the down factor $d = 0.8$, $T = 1$ year and each period being of length six months. The risk-free interest rate is 5% per annum with continuous compounding. Construct the two-period binomial tree for the stock. Find the price of an American put option with strike K = ₹104 and maturity $T = 1$ year. ($e^{-0.025} = 0.9753$)

5. (a) Given that in a risk-neutral world,

$$\ln S_T \sim \phi\left[\ln S_0 + \left(r - \frac{\sigma^2}{2}\right)T, \sigma^2 T\right],$$

where S_T is the stock price at a future time T, S_0 is the current stock price, r is the risk-free rate, σ is the volatility and $\phi(m, v)$ denotes a normal distribution with mean m and variance v. For the given strike price K, find $P(S_T > K)$, the probability that a European call option be exercised in a risk-neutral world.

(b) Show that the Black—Scholes-Merton formulas for call and put options satisfy the put-call parity.

(c) What is the price of a European call option on a non-dividend-paying stock when the stock price is ₹52, the strike price is ₹50, the risk-free interest rate is 12% per annum, the volatility is 30% per annum, and the time to maturity is three months?

(You can use values: $\ln(26/25) = 0.0392$, $\exp(-0.03) = 0.9704$)

6. (a) Discuss the delta of a European call option and calculate the delta of an at-the-money 6-month European call option on a non-dividend-paying stock when the risk-free interest rate is 8% per annum and the stock price volatility is 30% per annum.

(b) Companies A and B have been offered the following rates per annum on a ₹10 million loan for 5 years :

	Fixed rate	Floating rate
Company A	12.0%	LIBOR+ 0.1%
Company B	14.5%	LIBOR + 0.9%

Company A requires a floating-rate loan; Company B requires a fixed-rate loan. Design a swap that will net a bank, acting as intermediary, 0.1% per annum and that will appear equally attractive to both companies.

P.T.O.

(c) Find the payoff from a bull spread created using call options. Also draw the profit diagram corresponding to this trading strategy.

[This question paper contains 4 printed pages.]

Your Roll No..............

Sr. No. of Question Paper : 3182 **H**

Unique Paper Code : 32357610

Name of the Paper : DSE-4(i): Number Theory

Name of the Course : **B.Sc. (H) Mathematics**

Semester : VI

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.

2. **All** questions are compulsory.

3. Attempt any **two** parts from each question.

1. (a) A farmer purchased 100 head of livestock for a total cost of Rs. 4000. Prices were as follow: calves, Rs.120 each; lambs, Rs.50 each; piglets, Rs.25 each. If the farmer obtained at least one animal of each type, how many of each did he buy? (6.5)

(b) Define a complete set of residues modulo n. Verify that 0, 1, 2, 2^2, 2^3, ⋯, 2^9 form a complete set of residues modulo 11, but that 0, 1^2, 2^2, 3^2, ⋯ 10^2 do not. (6.5)

(c) Obtain three consecutive integers, the first of which is divisible by a square, the second by a cube, and the third by a fourth power. (6.5)

2. (a) State and prove Wilson's theorem. What about its converse? Justify your answer. (6.5)

(b) (i) Use Fermat's theorem to show that if p is an odd prime, then
$1^p + 2^p + 3^p + \cdots + (p-1)^p \equiv 0 \pmod{p}$.

(ii) If p and q are distinct primes, prove that
$p^{q-1} + q^{p-1} \equiv 1 \pmod{pq}$. (6.5)

(c) Define Mobius μ-function. If the integer n > 1 has the prime factorization $n = p_1^{k_1} p_2^{k_2} \cdots p_2^{k_r}$ then prove that

$$\sum_{d|n} \frac{\mu(d)}{d} = \left(1 - \frac{1}{p_1}\right)\left(1 - \frac{1}{p_2}\right) \cdots \left(1 - \frac{1}{p_r}\right).$$ (6.5)

3. (a) Find the highest power of 5 and the highest power of 2 in 1000! and hence find the number of zeros with which the decimal representation of 1000! terminates. (6.5)

(b) Prove that for $n > 1$, the sum of the positive integers less than n and relatively prime to n is $\frac{1}{2}n\phi(n)$ and hence find the sum of the positive integers less than 100 and relatively prime to 100. (6.5)

(c) Define Euler's ϕ-function. Show that for each positive integer $n \geq 1$,

$$n = \sum_{d|n} \phi(d)$$

the sum being extended over all positive divisors of n. Verify this result for n=32. (6.5)

4. (a) Show that if $\gcd(m,n) = 1$, where $m > 2$ and $n > 2$, then the integer mn has no primitive roots. Hence deduce that 21 has no primitive roots. (6.5)

(b) Define primitive root of the integer $n > 1$. Find all the primitive roots of 25. (6.5)

(c) State Euler's criterion to determine whether an integer a is a quadratic residue of a given prime p. Also show that 3 is a quadratic residue of 13 but a quadratic nonresidue of 17. (6.5)

5. (a) Show that if r is a primitive root of the prime $p \equiv 1 \pmod 4$, then $r^{(p-1)/4}$ satisfies the quadratic congruence $x^2 + 1 \equiv 0 \pmod p$. (6.5)

(b) Obtain the solution of the quadratic congruence

$$x^2 \equiv 23 \pmod{7^2}$$ (6.5)

(c) Let p be an odd prime and let a and b be integers that are relatively prime to p. Then prove that $(ab/p) = (a/p)(b/p)$.

Further determine whether the congruence $x^2 \equiv -46 \pmod{17}$ is solvable. (6.5)

6. (a) When the RSA algorithm is based on the key $(n,k) = (1537,47)$, what is the recovery exponent for the cryptosystem? (5)

(b) Decrypt the message HOZTKGH, which was produced using the linear cipher $C \equiv 3P + 7 \pmod{26}$. (5)

(c) Use the Hill's cipher

$$C_1 \equiv 5P_1 + 2P_2 \pmod{26}$$
$$C_2 \equiv 3P_1 + 4P_2 \pmod{26}$$

to encipher the message GIVE THEM TIME. (5)

[This question paper contains 8 printed pages.]

Your Roll No...............

Sr. No. of Question Paper : 3183 **H**

Unique Paper Code : 32357616

Name of the Paper : DSE-4 Linear Programming and Applications

Name of the Course : **CBCS (LOCF) – B.Sc. (H) Mathematics**

Semester : VI

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt any **two** parts from each questions.
3. **All** questions carry equal marks.

1. (a) Define a Convex Set. Show that the set S defined as :

$$S = \{(x, y) \mid y^2 \leq 4x\} \text{ is a Convex Set.}$$

P.T.O.

(b) Let $x_1 = 1$, $x_2 = 2$, $x_3 = 4$ be a feasible solution to the system of equations :

$$2x_1 + 3x_2 - x_3 = 4$$

$$3x_1 - x_2 + x_3 = 5$$

Is this a basic feasible solution? If not, reduce it to two different basic feasible solutions.

(c) Consider the following linear programming problem:

Minimize $z = cx$

subject to $Ax = b$, $x \geq 0$

Let $(x_B, 0)$ be a basic feasible solution with objective function value z_B corresponding to a basis B where $x_B = B^{-1}b$. By entering an a_j with $z_j - c_j > 0$ and removing a b_r subject to :

$$\frac{x_{Br}}{y_{rj}} = \text{Min}\left[\frac{x_{Bi}}{y_{ij}} : y_{ij} > 0\right]$$

Show that we can get a new feasible solution with improved value of the objective function compared to z_B.

2. (a) Using Simplex method, find the solution of the following linear programming problem :

Maximize $z = x_1 - 2x_2 + x_3$

subject to $x_1 + 2x_2 + x_3 \leq 12$

$2x_1 + x_2 - x_3 \leq 6$

$x_1 - 3x_2 \geq -9$

$x_1, x_2, x_3 \geq 0.$

(b) Using two phase method, solve the linear programming problem :

Minimize $z = -3x_1 + x_2$

subject to $2x_1 + x_2 \geq 2$

$x_1 + 3x_2 \leq 3$

$x_2 \leq 4$

$x_1, x_2 \geq 0.$

(c) Solve the following linear programming problem by Big - M method :

Maximize $z = 3x_1 + 2x_2 + 3x_3$

subject to $2x_1 + x_2 + x_3 \leq 2$

$3x_1 + 4x_2 + 2x_3 \geq 8$

$x_1, x_2, x_3 \geq 0.$

3. (a) Consider the following primal problem (P) and dual problem (D) :

(P) Minimize $z = cx$

Subject to $Ax \geq b,\ x \geq 0$

(D) Maximize $z = wb$

Subject to $wA \leq c,\ w \geq 0,$

If x_0 (w_0) is an optimal solution to the primal (dual) problem then there exists a feasible solution $w_0(x_0)$ to the dual (primal) such that $cx_0 = w_0b$.

(b) Use graphical method to solve the dual of the following linear programming problem :

Minimize $z = 6x_1 + 8x_2 + 7x_3 + 15x_4$

Subject to $x_1 + x_3 + 3x_4 \geq 4$

$x_2 + x_3 + x_4 \geq 3$

$x_1, x_2, x_3, x_4 \geq 0$

Further, find an optimal solution to the given problem from optimal solution of the dual problem.

(c) Obtain the dual of the following linear programming problem :

Maximize $z = 10x_1 + x_2 + 2x_3$

Subject to $x_1 + x_2 - 2x_3 \geq 10$

$x_1 + 4x_2 - 3x_3 = 3$

$4x_1 + x_2 + x_3 \leq 20$

$x_1 \geq$, $x_2 \leq 0$, and x_3 unrestricted in sign.

4. (a) A Company has four warehouses, a, b, c and d. It is required to deliver a product from these warehouses to three customers A, B and C. The warehouses have the following amounts in stock.

Warehouse :	a	b	c	d
No. of units :	15	16	12	13

and the customer's requirements are

Customers :	A	B	C
No. of units	18	20	18

The table below shows the costs of transporting one unit from warehouses to the customers :

	a	b	c	d
A	8	9	6	3
B	6	11	5	10
C	3	8	7	9

Find the optimal schedule and minimum total transport cost.

(b) A Company is faced with the problem of assigning six different machines to six different jobs. Determine the optimal solution of the Assignment Problem with the following cost matrix :

	a	b	c	d	e	f
1	9	22	58	11	19	27
2	43	78	72	50	63	48
3	41	28	91	37	45	33
4	74	42	27	49	39	32
5	36	11	57	22	25	18
6	3	56	53	31	17	28

(c) For the following cost minimization Transportation Problem, find initial basic feasible solution by using North-West comer rule, Least Cost method and Vogel's approximation method. Compare the three solutions (in terms of cost).

	A	B	C	D	Supply
I	19	14	23	11	11
II	15	16	12	21	13
III	30	25	16	39	19
Demand	6	10	12	15	

5. (a) Define the Saddle point. The pay-off matrix of a game is given below. Find the best strategy for each player, and the value of a play of the game of A and B.

Player B

Player A		I	II	III	IV	V
	I	9	3	1	8	0
	II	6	5	4	6	7
	III	2	4	3	3	8
	IV	5	6	2	2	1

(b) Convert the following Game problem into a linear programming problem for Player A and Player B and solve it by Simplex method.

	Player B		
Player A	3	-2	4
	-1	4	2

(c) Using Simplex method, solve the system of equations :

$$3x_1 + x_2 = 7$$

$$x_1 + x_2 = 3$$

Also write the inverse of the matrix $\begin{bmatrix} 3 & 1 \\ 1 & 1 \end{bmatrix}$.

[This question paper contains 8 printed pages.]

Your Roll No..............

Sr. No. of Question Paper : 2301 **H**

Unique Paper Code : 62354443

Name of the Paper : Analysis (LOCF)

Name of the Course : **B.Sc. (Prog.)**

Semester : IV

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.

2. **All** questions are compulsory.

3. Attempt any **two** parts from each question.

4. **All** questions carry equal marks.

P.T.O.

1. (a) State and prove Archimedean property of real numbers.

(b) Let A and B be two nonempty bounded sets of positive real numbers, and let

$$C = \{xy : x \in A \text{ and } y \in B\}.$$

Show that C is bounded and :

(i) $\text{Sup } C = \text{Sup } A \text{ Sup } B$

(ii) $\text{Inf } C = \text{Inf } A \text{ Inf } B$

(c) Let f be a function on $\mathbb{R}$ defined by

$$f(x) = \begin{cases} 1, & \text{when x is rational} \\ -1, & \text{when x is irrational} \end{cases}$$

Show that f is discontinuous at every point of $\mathbb{R}$.

(d) Define Uniform continuity of a function f on an interval I. Show that the function f defined by $f(x) = \sqrt{x}$ is uniformly continuous in the interval [1,3].

2. (a) Show that the sequence $<r^n>$ converges to zero if $|r| < 1$. Discuss other cases also.

(b) Show that $\lim_{n\to\infty} \sqrt[n]{n} = 1$.

(c) Define the limit of a sequence of real number. Let $<a_n>$ be a sequence of positive terms such that $<a_n> \to a$. Then prove that $\sqrt{a_n} \to \sqrt{a}$.

(d) If f and g be two real functions defined on some neighbourhood of c such that $\lim_{x\to c} f(x) = l$,

$\lim_{x \to c} g(x) = m$, then show that

$$\lim_{x \to c} (fg)x = \lim_{x \to c} f(x) \lim_{x \to c} g(x) = lm.$$

3. (a) Let $<a_n>$ be a sequence of real numbers such that $a_n \neq 0$ for all n and

$$\lim_{n \to \infty} \left(\frac{a_{n+1}}{a_n} \right) = l, \quad \text{where } |L| < 1.$$

Show that $\lim_{n \to \infty} a_n = 0$. Also, Deduce that

$$\lim_{n \to \infty} 2^{-n} n^2 = 0.$$

(b) Define Cauchy sequence. Use Cauchy's General Principle of Convergence to show that the sequence $<a_n>$ defined by

$$a_n = 1 + \frac{1}{3} + \frac{1}{5} + \cdots + \frac{1}{2n-1}$$

does not converge.

(c) Prove that the sequence $<S_n>$ defined by the recursion formula : $S_{n+1} = \sqrt{3S_n}$, $S_1 = 1$, $n \geq 2$ converges to 3.

(d) Give an example of a bounded subset $S \neq \phi$ of $\mathbb{R}$ whose least upper bound and greatest lower bound belong to S^c.

4. (a) State Cauchy's n^{th} root test for the series. Use this test to check the convergence of the series

$$\frac{n^{n^2}}{(n+1)^{n^2}}$$

(b) State and prove the necessary condition for the convergence of a series. Is the converse holds true? Justify your answer.

(c) Test the convergence of the following series

(i) $\sum_{n=1}^{\infty} \frac{r^n}{n!}$ where r is any positive number.

(ii) $\sum_{n=1}^{\infty} \frac{n^n x^n}{n!} \; (x > 0)$

(d) State the D' Alembert ratio test and Raabe's test for the convergence of the series. Test the convergence of the series

$$\sum_{n=1}^{\infty} \frac{(n!)^2}{(2n)!} x^n$$

5. (a) Define continuity of a real valued function at a point.

Show that the function defined as $\begin{cases} \frac{x^2-9}{x-3}, & x \neq 3 \\ 6, & x = 3 \end{cases}$

is continuous at x=3.

(b) Show that every absolutely convergent series is convergent. Is the converse holds true? Give example.

(c) Show that the function f defined on [a, b] as

$$f(x)=\begin{cases}0 & \text{when x is rational}\\ 1 & \text{when x is irrational}\end{cases}$$

is not Riemann integrable.

(d) Show that the series $1 + r + r^2 + r^3 + \cdots$ $(r > 0)$ converges if $r < 1$ and diverges if $r > 1$.

6. (a) Define Riemann integrability of a bounded function f on a bounded closed interval [a,b],

(b) Use the definition of Riemann integrability to prove

$$\int_0^b x^2\,dx = \frac{b^3}{3}$$

P.T.O.

(c) State Non-uniform Continuity Criteria of a function defined on $A \subseteq R$. Use it to prove $f(x) := \frac{1}{x}$ is not uniform continuous on $A = \{x \in R \mid x > 0\}$.

(d) Define supremum and infimum of the set $S \subseteq \mathbb{R}$. Find the supremum and infimum of the set

$$S = \{1 - (-1)^n/n : n \in \mathbb{N})$$

[This question paper contains 4 printed pages.]

Your Roll No..............

Sr. No. of Question Paper : 2363 **H**

Unique Paper Code : 62354443

Name of the Paper : Analysis

Name of the Course : **B.A. (Programme) – Core Course**

Semester : IV

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt **all** question by selecting **two** parts from each question.
3. Part of questions to be attempted together.
4. **All** questions carry equal marks.
5. Use of calculator not allowed.

1\. (a) Define the upper bound and lower bound of a non-empty subset of $\mathbb{R}$. Find the upper bound and lower bound for the following sets, if they exist. (6.5)

(i) $\{-1, 1, -2, -2, \ldots, -n, n \ldots\}$

P.T.O.

(ii) $\left\{0, \frac{1}{3}, \frac{2}{4}, \ldots, \frac{n-1}{n+1}, \ldots\right\}$

(iii) $\left\{2, \frac{3}{2}, \frac{4}{3}, \ldots, \frac{n+1}{n}, \ldots\right\}$

(iv) $\{1^2, 2^2, 3^2, \ldots, n^2, \ldots\}$

(b) Define continuity of a real valued function at a point. Show that the function defined as

$$f(x) = \begin{cases} \frac{x^2-9}{x-3}, & x \neq 3 \\ 6, & x = 3 \end{cases}$$

is continuous at $x = 3$. (6.5)

(c) State and Prove the Archimedean property of real numbers. (6.5)

2. (a) Show that the function $f(x) = x^2$ is uniformly continuous in the interval $[-2, 2]$. (6)

(b) Define the Sequential Criterion of limit. Show that $\lim_{x \to 0} \sin\left(\frac{1}{x}\right)$ does not exist in $\mathbb{R}$. (6)

(c) State the Sequential Criterion of continuity. Define $f: \mathbb{R} \to \mathbb{R}$ by

$$f(x) = \begin{cases} 1, & \text{if } x \text{ is rational} \\ 0, & \text{if } x \text{ is irrational} \end{cases}$$

Show that f is not continuous at any point of $\mathbb{R}$. (6)

3. (a) Use the definition of limit of a sequence to establish the following limits :

(i) $\lim_{n\to\infty}\left(\frac{n}{n^2+1}\right)=0$. (3.5)

(ii) $\lim_{n\to\infty}\left(\frac{3n+1}{2n+5}\right)=\frac{3}{2}$. (3)

(b) Discuss the convergence of the sequence $\left(\frac{11^{2n}}{7^{3n}}\right)$. (6.5)

(c) Show that the sequence (x_n) defined by

$$x_1 = a > 0,\ x_{n+1} = \frac{2x_n}{1+x_n},\quad n > 1,$$

is bounded and monotone. Also, find the limit. (6.5)

4. (a) Give an example of an unbounded sequence that has a convergent subsequence. (6)

(b) Show that the sequence (x_n) defined by

$$x_n = 1+\frac{1}{6}+\frac{1}{11}+\cdots+\frac{1}{5n-4}$$

is not Cauchy. (6)

(c) Calculate the value of $\sum_{n=2}^{\infty}\left(\frac{2}{7}\right)^n$. (6)

5. (a) State D'Alembert's ratio test for an infinite series. Test for convergence of the series

$$\sum_{n=1}^{\infty} \frac{2^n n!}{n^n}. \qquad (6.5)$$

(b) Test for convergence the following series :

(i) $\sum_{n=1}^{\infty}\left(1+\frac{1}{\sqrt{n}}\right)^{-n^{\frac{3}{2}}}$ (3.5)

(ii) $\frac{1^2.2^2}{1!}+\frac{2^2.3^2}{2!}+\frac{3^2.4^2}{3!}+\cdots$ (3)

(c) Show that the greatest integer function f(x) = [x] is Riemann integrable on [0, 4] and $\int_0^4 [x]dx = 6$. (6.5)

6. (a) Test the convergence and absolute convergence of the series :

(i) $\sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{n\sqrt{n}}$ (3)

(ii) $\frac{1}{1.2}-\frac{1}{3.4}+\frac{1}{5.6}-\frac{1}{7.8}+\cdots$. (3)

(b) State Leibnitz's test for an alternating series. Test for convergence the series :

$$1-\frac{1}{\sqrt{3}}+\frac{1}{\sqrt{5}}-\frac{1}{\sqrt{7}}+\cdots \qquad (6)$$

(c) Show that every monotonic function on [a, b] is Riemann integrable on [a, b]. (6)

[This question paper contains 8 printed pages.]

Your Roll No...............

H

Sr. No. of Question Paper : 4986

Unique Paper Code : 2342202402

Name of the Paper : Data Mining II

Name of the Course : **B.A. (P) (NEP)**

Semester : IV

Duration : 3 Hours　　　　Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.

2. **Section A** is compulsory.

3. Attempt any **four** questions from **Section B.**

4. Parts of a question must be answered together.

5. Use of scientific calculator is allowed.

Section A

1. (a) How does the number of clusters affect anomaly detection in k-means clustering algorithm? (2)

P.T.O.

(b) In a dataset of monthly sales figures for a retail store, the mean monthly sales are Rs. 50,000 with a standard deviation of Rs. 5,000. In a certain month, the store recorded sales of Rs. 65,000. Calculate the z-score for this month's sales. (2)

(c) Consider a dataset with binary labels. The dataset is trained using Adaboost method. The decision boundary obtained after a single iteration is shown in figure II. (3)

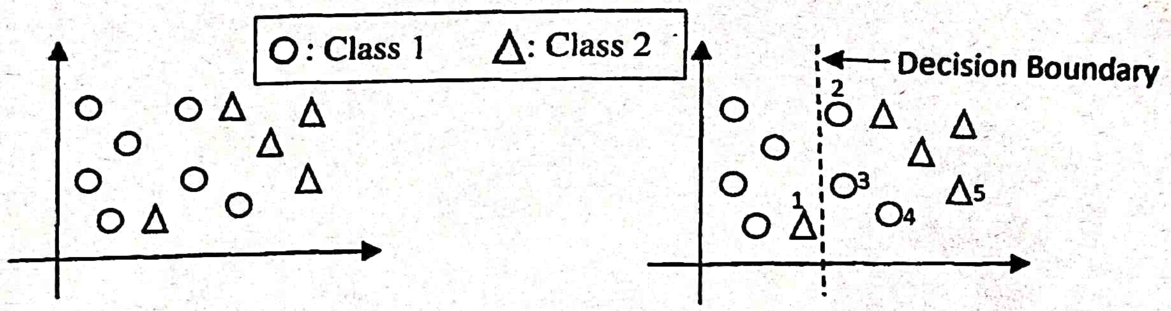


Figure I: Original Dataset

Figure II: Classification after 1st iteration

In figure II, out of the points marked 1, 2..., 5, which points shall have higher weights? Justify your answer.

(d) What is overfitting in the context of classification? Name two methods to prevent it. (3)

(e) Can clustering be used for dimensionality reduction? Justify your answer. (3)

(f) What is downsampling? Perform downsampling on the data given below : (3)

Class 0	9000 samples
Class 1	1000 samples

(g) Discuss any two key issues and their solutions in hierarchical clustering. (4)

(h) How does a proximity matrix differ from data matrix and why is it advantageous for anomaly detection methods? (4)

(i) Consider the following dataset with two documents related to customer's reviews of a product:

Document 1: "The product is excellent. I am very satisfied with its performance."

Document 2: "This product exceeded my expectations. It works flawlessly."

Perform the following tasks :

(i) Calculate the Term Frequency (TF) for the term "product" in each document.

(ii) Compute the Inverse Document Frequency (IDF) for the term "product" in the given document corpus.

(iii) Apply Frequency Damping with a damping parameter k=0.2 to the TF values obtained in part (i) for the term "product" in each document. (6)

Section B

2\. (a) Consider the following distance matrix for the five data points P1 to P5. (7)

	P1	P2	P3	P4	P5
P1	0				
P2	0.25	0			
P3	0.23	0.17	0		
P4	0.38	0.21	0.17	0	
P5	0.36	0.15	0.27	0.30	0

Perform hierarchical clustering using centroid linkage on the given distance matrix and show the dendrogram.

(b) Consider the following data set comprising of eight points A1, A2, ..., A8 on a plane. (8)

A1(2, 10), A2(2, 5), A3(8, 4), A4(5, 8), A5(7, 5), A6(6, 4), A7(1, 2), A8(4, 9)

K-means clustering algorithm is applied to find the three clusters by keeping A1(2, 10), A4(5, 8) and A7(1, 2) as initial cluster centers. Find the cluster centres obtained in the next iteration. Use Manhattan distance to find distance/similarity among data points. Show all the intermediate steps.

3. (a) What is the primary objective of time series analysis in data mining? Consider the following time series dataset containing daily temperature fluctuation over a week: (7)

Monday: 25°C

Tuesday: 24°C

Wednesday: 22°C

Thursday: 26°C

Friday: 28°C

Saturday: 29°C

Sunday: 27°C

If the dataset is shifted by two days forward, what will be the temperature on Tuesday of the second week?

(b) What is the significance of the Bagging algorithm in ensemble learning? How does Bagging combine multiple weak learners to create a strong classifier, illustrate with the help of a diagram. Suppose we have a dataset with the following features and labels :

Features (X)	1	2	3	4	5	6	7	8	9	10
Labels (Y)	1	0	1	1	0	0	1	0	1	0

P.T.O.

Generate two sub datasets containing four samples each from the original dataset using bootstrap sampling method with replacement and two sub datasets without replacement. (8)

4. (a) Discuss the relationship between entropy and information content and provide mathematical formula to calculate entropy. (9)

Consider the following dataset of 10 instances labelled as either "Positive" or "Negative", calculate entropy on this dataset.

X	Label
1	Positive
2	Negative
3	Positive
4	Positive
5	Negative
6	Positive
7	Negative
8	Negative
9	Positive
10	Positive

(b) Compare the following anomaly detection techniques :– (6)

(i) Model-Based and Model-Free method

(ii) Label and Score method

(iii) Global and Local Perspective method

5. (a) Explain the E-step and M-step in the Expectation-Maximization (EM) algorithm and (9) Probabilistic Latent Semantic Analysis (PLSA). How are the parameters of PLSA updated in each step? Illustrate the generative processes of EM-clustering and PLSA through appropriate diagrams. (9)

(b) Consider the following text in a document D1:

Document D1: "Text mining is a technique used to extract useful information from text documents,"

Perform the following text mining preprocessing steps on the text given and write your answer:

(i) Stop Word Removal

(ii) Stemming

(iii) Removal of punctuation marks (6)

6. (a) What is the principle of the STREAM algorithm? How does it handle data streams effectively? (5)

(b) Consider the following stream of data points arriving with timestamps. The data points are as follows : (10)

Time Stamp	Data Point
0	(2,3)
1	(3,4)
2	(5,6)
3	(8,9)
4	(10,12)

Apply the STREAM algorithm with a window size of 2 and determine the micro-clusters formed at the end of the data stream.

7. (a) Describe the steps of a random forest classifier. Discuss the impact of attribute selection at internal nodes on improving the classifier's performance. (5)

(b) Consider the following dataset of six data points :

A(1, 2), B(2, 3), C(3, 4), D(7, 8), E(8, 9), F(25, 30)

Apply the Density-Based Spatial- Clustering of Applications with Noise (DBSCAN) algorithm on the given data with Epsilon (ε) = 2.5 units, and Minimum Points (minPts)= 2. Identify the clusters, core points, reachable points and noise points. (10)

[This question paper contains 4 printed pages.]

Your Roll No...............

Sr. No. of Question Paper : 3365 **H**

Unique Paper Code : 42354401

Name of the Paper : Real Analysis

Name of the Course : **B.Sc. (Prog) Physical Sciences / Mathematical Sciences**

Semester : IV

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.

2. **All** questions are compulsory.

3. Attempt any **two** parts from each question.

1. (a) Let A and B be non- empty bounded subsets of R. Let $A + B = \{a + b: a \in A, b \in B\}$.

Prove that $\inf (A+B) = \inf A + \inf B$.

(b) Define denumerable set. Show that the set $N \times N$ is a denumerable set.

P.T.O.

(c) Let S be a bounded set in R and let S_0 be a non-empty subset of S. Show that $\inf S \le \inf S_0 \le \sup S_0 \le \sup S$. (6,6)

2. (a) (i) Define the convergence of a sequence (x_n) of real numbers.

(ii) Let (x_n) be a sequence of real numbers that converges to x. Show that the sequence $(|x_n|)$ of absolute values converges to $|x|$.

(b) Prove that if $c > 0$, then $\lim_{n\to\infty} c^{1/n} = 1$.

(c) Show that if $X = (x_n)$ and $Y = (y_n)$ are sequences of real numbers converging to x and y respectively, then their product $X \cdot Y$ converges to xy. (6.5,6.5)

3. (a) State Monotone Convergence Theorem. Show that the sequence (x_n) defined by

$x_1 = 1;\ x_{n+1} = \sqrt{2 + x_n},\ \forall n \in N$ is convergent.

Also, find $\lim_{n\to\infty} x_n$.

(b) Define subsequence of a sequence (x_n) of real numbers. Prove that if every subsequence of $X = (x_n)$ has a subsequence that converges to 0, then $\lim X = 0$.

(c) Prove that a Cauchy sequence of real numbers is bounded. Is the converse true? Justify your answer. (6.5,6.5)

4. (a) State and prove Limit Comparison Test for positive term series. Hence, show that the following series converges :

$$\frac{\sqrt{2}}{3.5}+\frac{\sqrt{4}}{5.7}+\frac{\sqrt{6}}{7.9}+\frac{\sqrt{8}}{9.11}+\cdots$$

(b) Check the convergence of the following series :

(i) $\sum_{n=1}^{\infty}\sin\left(\frac{n\pi}{2}\right)$

(ii) $\sum_{n=1}^{\infty}(-1)^{n}\left(\frac{n}{2n-1}\right)$

(iii) $\sum_{n=1}^{\infty}\left(n^{\frac{1}{n}}-1\right)^{n}$

(c) State and prove Cauchy's n^{th} Root test for a series of non-negative real numbers. (6,6)

5. (a) Prove that an absolutely convergent series is convergent. Is the converse true? Justify your answer.

(b) Define a power series. Determine the radius of convergence and exact interval of convergence of the power series $\sum \frac{x^n}{n^n}$.

(c) State Weierstrass M-Test for uniform convergence of series. Hence, show that

$$\sum \frac{1}{n^3 + n^4 x^4}, \quad \forall x \in R$$

is uniformly convergent. (6.5,6.5)

6. (a) Show that $f(x) = k \ \forall x \in [a, b]$ is Riemann integrable, where k is a constant.

(b) Show that the sequence $\left(\frac{x^2 + nx}{n}\right)$ is pointwise convergent but not uniformly convergent on R.

(c) Find the upper and lower Darboux integrals for $f(x) = x^3$ on the interval $[0, b]$. Is f integrable on $[0, b]$? (6,6)

[This question paper contains 8 printed pages.]

Your Roll No..............

Sr. No. of Question Paper : 3472 H

Unique Paper Code : 42357618

Name of the Paper : Numerical Methods

Name of the Course : B.Sc. (Prog.) Physical Sciences / Mathematical Sciences

Semester : VI

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.

2. Attempt any two parts from each question.

3. All questions carry equal marks.

P.T.O.

1. (a) Define the local truncation error. Round off the number 754682 to four significant digits and then calculate the absolute, relative and percentage errors. (6.25)

(b) Using the Regula Falsi method, compute the real root of the equation $x^3 = 6$ correct to four decimal places. (6.25)

(c) Perform four iterations of the Bisection method to obtain the smallest positive root of the equation $f(x) = x^3 - 4x + 1 = 0$. (6.25)

2. (a) Compute $\sqrt{5}$ correct to four decimal places by using Newton-Raphson Method. (6.25)

(b) Solve the linear system $AX = b$, where

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 10 \\ 3 & 14 & 28 \end{bmatrix}, \; b = \begin{bmatrix} 1 \\ -2 \\ -8 \end{bmatrix}$$

by Gauss Elimination method using partial pivoting.

(6.25)

(c) Using Secant method, find the smallest positive root of the equation $x^4 - x = 10$ correct to three decimal digits. (6.25)

3. (a) Perform three iterations of Gauss-Seidel method to solve the linear system

$$2x - y + 0z = 7$$

$$-x + 2y - z = 1$$

$$0x - y + 2z = 1$$

Take the initial approximation as (x, y, z) = (0, 0, 0).

(6.25)

(b) Find the inverse of the matrix A using Gauss-Jordan method, where

$$A = \begin{bmatrix} 3 & 2 & 1 \\ 2 & 3 & 2 \\ 1 & 2 & 2 \end{bmatrix}$$ (6.25)

(c) Consider the following table :

x	0.1	0.2	0.3	0.4	0.5
$f(x)$	1.40	1.56	1.76	2.00	2.28

Use Newton divided difference formula to calculate the interpolating polynomial and give an estimate for f(0.25). (6.25)

4. (a) Obtain the piecewise quadratic interpolating polynomial for

x	-2	-1	1	2	4
f(x)	-29	-8	-2	-5	7

Interpolate at x = 3.0. (6.25)

(b) Construct the interpolating polynomial by using Gregory-Newton backward difference interpolation formula for the given data :

x	1	1.5	2.0	2.5
f(x)	2.7183	4.4817	7.3891	12.1825

Estimate the value of f(2.25). (6.25)

(c) Construct the Richardson extrapolation table to find the derivative of $f(x) = 3^x$ at x = 3 using the central difference formula

$$f'(x) = \frac{f(x+h) - f(x-h)}{2h}$$

taking h = 1,2. (6.25)

5. (a) Consider the following table :

x	1	2	3	4	5
$f(x)$	2	4	8	16	32

Find $f'(3)$ using

(i) Central difference formula

(ii) Three-point forward difference formula.

(6.25)

(b) Compute the value of $\int_{-3}^{3} x^4\, dx$

using the Simpson's 1/3 rule taking six equal subintervals. (6.25)

(c) Apply trapezoidal rule to evaluate the integral

$$\int_0^6 \frac{dx}{1+x^2}$$

Calculate the difference between the exact value and the approximate value. (6.25)

6. (a) Apply Midpoint method to find approximate solution of the initial value problem

$$\frac{dy}{dx} = x + 2y, \quad y(0) = 0, \quad h = 0.1.$$ (6.25)

(b) Apply Euler's method to find approximate solution of the initial value problem

$$\frac{dy}{dx} = \frac{x-y}{z}, \quad y(0) = 1, \quad 0 \le x \le 3, \quad h = 0.5.$$ (6.25)

(c) Find an approximation to $y(0.2)$, for the initial value problem

$$\frac{dy}{dx} = y + x, \quad y(0) = 1, \quad 0 \leq x \leq 0.2.$$

using the Heun's method with $h = 0.1$. Compare with exact solution. (6.25)

[This question paper contains 8 printed pages.]

Your Roll No..............

Sr. No. of Question Paper : **4020** **H**

Unique Paper Code : 2352571201

Name of the Paper : Elementary Linear Algebra

Name of the Course : **B.A. / B.Sc. (Prog.) with Mathematics as Non-Major/ Minor – DSC**

Semester : II

Duration : 3 Hours Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.

2. Attempt **all** questions by selecting any **two** parts from each question.

3. **All** questions carry equal marks.

4. Use of simple calculator is allowed.

P.T.O.

1 (a) If x and y are vectors in $\mathbb{R}^n$, then prove that $||x + y|| \geq |\,||x|| - ||y||\,|$. Also verify it for the vectors x = [2, –1, 3, 2] and y = [4, 3, 2 1] in $\mathbb{R}^4$. (5.5+2)

(b) Solve the following system of linear equations using Gaussian Elimination method. Also indicate whether the system is consistent or inconsistent.

$$3x - 2y + 4z = -54$$

$$-x + y - 2z = 20$$

$$5x - 4y + 8z = -83$$ (6.5+1)

(c) Find the quadratic equation $y = ax^2 + bx + c$ that goes through the points (3,18), (2,9) and (–2,13). (7.5)

2. (a) Solve the following system of equations using the Gauss-Jordan method :

$$5x + 20y - 18z = -11$$

$$3x + 12y - 14z = 3$$

$$-4x - 16y + 13z = 13$$

(7.5)

(b) Find the rank of the following matrix by converting to row echelon form :

$$\begin{bmatrix} 8 & 0 & 0 & 1 \\ 1 & 0 & 3 & 1 \\ 0 & 0 & 1 & 3 \\ 0 & 8 & 1 & 8 \end{bmatrix}$$

(7.5)

(c) Find the characteristic polynomial, eigenvalues and corresponding eigenvectors for the given matrix :

$$\begin{bmatrix} -1 & 2 \\ 4 & -3 \end{bmatrix}$$

(2+2+3.5)

3. (a) Use the Diagonalization Method to determine whether the following matrix is diagonalizable.

$$\begin{bmatrix} 3 & 1 & -1 \\ 2 & 2 & -1 \\ 2 & 2 & 0 \end{bmatrix}$$ (7.5)

(b) Show that the set M_{22} of all 2×2 matrices is a vector space under the usual operations of addition and scalar multiplication. (7.5)

(c) Show that span (S) is a subspace of V, where S a nonempty subset of a vector space V. Let P_3 be the vector space of all real polynomials of degree ≤ 3 and $S = \{x, x^2 + 1, x^3 - 1\}$. Find span (S). Does $(x^2 + x^3) \in$ span (S)? (3+3+1.5)

4. (a) Check whether the following subset of R is linearly independent or not.

$$S = \{(0,1,-1), (1,1,0), (1,0,2)\}.$$

Express (1,1,1) as linear combination of vectors in S. (4+3.5)

(b) Define a basis of a vector space. Show that the following set S is a basis for the vector space M_{22} of all 2 × 2 matrices :

$$S = \left\{ \begin{pmatrix} 1 & 4 \\ 2 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 2 \\ 1 & 0 \end{pmatrix}, \begin{pmatrix} -3 & 1 \\ -1 & 0 \end{pmatrix}, \begin{pmatrix} 5 & -2 \\ 0 & -3 \end{pmatrix} \right\}$$ (2+5.5)

(c) Define a finite dimensional vector space. Let W be the solution set to the matrix equation $AX = 0$, where $A = \begin{pmatrix} 1 & -2 & 1 \\ 0 & 1 & 1 \end{pmatrix}$. Show the following :

(i) W is a subspace of R^3.

(ii) Find a basis for W. (1.5+3+3)

5. (a) Check if the following mappings are linear transformation or not. Prove it or give a counter example to disprove.

(i) $f: R^3 \to R^3$ defined as $f([a, b, c]) = [-b, c, 0]$

(ii) $g: M_{22} \to R$ defined as $g\left(\begin{bmatrix} a & b \\ c & d \end{bmatrix}\right) = ad - bc$,

where M_{22} is the vector space of 2×2 real matrices. (4+3.5)

(b) Find the matrix T_{AB} of linear transformation $T: R^2 \to R^3$ with respect to basis $B = \{(1,0), (0,1)\}$ and $C = \{(1,1,0), (0,1,1), (1,0,1)\}$, where

$$T([a,b]) = [a - b, a, 2a + b] \qquad (7.5)$$

(c) Define kernel of a linear transformation T. Show that $Ker(T) = \{0\}$ if and only if T is one-one. (2+5.5)

6. (a) Consider the linear operator $L : M_{33} \to M_{33}$, defined as $L(A) = A - A^T$, where M_{33} is vector space of 3×3 matrices and A^T denotes the transpose of the matrix A.

Find dim(Ker(L)) + dim(Range(L)). (7.5)

(b) Define an onto linear transformation. If T be a linear transformation from a finite dimensional vector space V to a finite dimensional vector space W, show that T is onto if and only if dim (Range(L)) = dim (W). (2+5.5)

(c) Show that the mapping f: $M_{nm} \rightarrow M_{mn}$ defined as $f(A) = A^T$ is an isomorphism. (7.5)

Sr. No. of Question Paper: 4965
Unique Paper Code: 2352571201
Name of the Paper: Elementary Linear Algebra
Type of the Paper: DSC
Semester: II
Programme: BA/B.Sc. (Prog.) with Mathematics as Non-Major/ Minor
Duration: 03 hours
Maximum Marks: 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt all questions by selecting any two parts from each question.
3. All questions carry equal marks.
4. Use of simple calculator is allowed.

1. (a) If x and y are vectors in $\mathbb{R}^n$, then prove that $\|x+y\| \leq \|x\| + \|y\|$. Also verify it for the vectors $x = [1, 3, 2, -1]$ and $y = [1, -2, 0, 3]$ in $\mathbb{R}^4$. (5.5+2)

(b) Solve the following system of linear equations using Gaussian Elimination method. Also indicate whether the system is consistent or inconsistent.

$$\begin{aligned} x + y + z &= 5 \\ 2x + y - z &= 2 \\ 2x - y + z &= 2 \end{aligned}$$

(6.5+1)

(c) Define equivalent systems. Also check whether the following system of equations are equivalent or not?

$$\begin{aligned} 2x - y &= 1 \\ 3x + y &= 9 \end{aligned} \quad \text{and} \quad \begin{aligned} x + 4y &= 14 \\ 5x - 2y &= 4 \end{aligned}$$

(2+5.5)

2. (a) Solve the following homogeneous system of equations using the Gauss-Jordan method:

$$\begin{aligned} -2x + y + 8z &= 0 \\ 7x - 2y - 22z &= 0 \\ 3x - y - 10z &= 0 \end{aligned}$$

(7.5)

(b) Define the rank of a matrix and hence find the rank of the following matrix by converting it to row echelon form:

$$\begin{bmatrix} 1 & 1 & -1 & 0 & 1 \\ 2 & -4 & 3 & 1 & 0 \\ 3 & 15 & -13 & -2 & 7 \end{bmatrix}$$

(2+5.5)

(c) Find the eigenvalues and their corresponding algebraic multiplicities for the given matrix:

$$\begin{bmatrix} 4 & 0 & -2 \\ 6 & 2 & -6 \\ 4 & 0 & -2 \end{bmatrix}$$

(7.5)

3. (a) Use the Diagonalization Method to determine whether the following matrix is diagonalizable.

$$\begin{bmatrix} -4 & 8 & -12 \\ 6 & -6 & 12 \\ 6 & -8 & 14 \end{bmatrix}$$

(7.5)

(b) (i) Determine whether the set of non-singular 2×2 matrices is a subspace of the vector space M_{22} of all 2×2 matrices.

(ii) Show that $S = \{(a, b, c) \in R^3 : 2a + b - c = 0\}$ is a subspace of R^3. (4+3.5)

(c) Define span of S, where S a nonempty subset of a vector space V. Determine span (S) where $S = \{(1,1,1),(2,1,0)\}$ is a subset of R^3. Also examine whether the following vectors of R^3 are in span (S):

(i) (0,0,0) (ii) (2,1,3). (2+3+1+1.5)

4. (a) Check whether the following subset of R^3 is linearly independent or not.

$S = \{(1,2,0),(0,3,1),(-1,0,1)\}$.

Express (1,2,1) as linear combination of vectors in S. (4+3.5)

(b) Define a finite dimensional vector space. Let W be the solution set to the matrix equation AX=0, where A= $\begin{pmatrix} -1 & 1 & 3 \\ 0 & 2 & -1 \end{pmatrix}$.

Show that (i) W is a subspace of R^3.

(ii) Find a basis for W. (2+3+2.5)

(c) (i) If v is linear combination of a vectors u_1, u_2, u_3 *and* u_4 in a vector space V, then show that $S = \{v, u_1, u_2, u_3, u_4\}$ is linearly dependent subset of V.

(ii) Show that any set containing 0 (zero vector) in a vector space is linearly dependent set. (4+3.5)

5. (a) Check if the following mappings defined on $P_n(x)$, real polynomials of degree at most n, are linear transformation or not. Prove it or give a counter example to disprove.

i. $f: P_n(x) \to R$ defined as $f(p(x)) = p(2)$.

ii. $g: P_n(x) \to R$ defined as $g(p(x)) = a$, where a is the coefficient of highest degree term. (4 + 3.5)

(b) Let $T : R^3 \to R^4$ be a linear transformation where $T([1,1,3]) = [2,3,4,1]$ and $T([3,2,1]) = [1,2,4,-1]$. Find $T([-1,0,5])$. (7.5)

(c) Define a linear transformation T from a vector space V to W. Further prove the following:

i. $T(0_v) = 0_w$, where 0_v and 0_w are the zero vectors in V and W respectively.

ii. $T(v - w) = T(v) - T(w)$. (1.5 + 3 + 3)

6. (a) Consider the linear transformation $L: P_2(x) \to P_4(x)$ defined as $L(p(x) = x^2 p(x))$. Find Range(L) and Ker(L). Further show $\dim(Ker(L)) = \dim(P_2(x)) - \dim(Range(L))$. (3 + 3+ 1.5)

(b) For the linear transformation $L : M_{23} \to M_{22}$ defined as:

$L\left(\begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix}\right) = \begin{bmatrix} a & -c \\ 2e & d+f \end{bmatrix}$, determine if it is one-one and onto. (4+3.5)

(c) For the linear transformation $L: R^3 \to R^3$ defined as, $L(v) = A\,v$, where

$A = \begin{pmatrix} 1 & 0 & 3 \\ 0 & 1 & 3 \\ 0 & 0 & -1 \end{pmatrix}$, determine L^{-1} if it exists. (7.5)

[This question paper contains 4 printed pages.]

Your Roll No..............

Sr. No. of Question Paper : 4902 **H**

Unique Paper Code : 2352572401

Name of the Paper : Abstract Algebra

Name of the Course : **B.A./B.Sc. (Programme) with Mathematics as Non-Major/ Minor – DSC**

Semester : IV

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt **all** questions by selecting **two** parts from each question.
3. Each part carries **7.5** marks.
4. Use of Calculator not allowed.

1. (a) State Division Algorithm. Determine 51 mod 13, 342 mod 85, 62 mod 15, (82 . 73) mod 7, (51+68) mod 7 and (35 . 24) mod 11.

P.T.O.

(b) Define a Group. Show that $G = \{1, -1, i, -i\}$ forms a group under complex multiplication.

(c) Let G be a group with the property that for any x, y, z in the group, $xy = zx$ implies $y = z$. Prove that G is Abelian. Also, in $GL(2, \mathbb{Z}_{13})$, find

$$\det\begin{bmatrix} 7 & 4 \\ 1 & 5 \end{bmatrix}.$$

2. (a) Let G be an abelian group and H and K be subgroups of G. Show that $HK = \{hk: h \in H, k \in K$ is a subgroup of G. Also, find the order of 7 in $\mathbb{Z}_{10}$ under addition modulo 10.

(b) Let a be an element in a group G of order 30. Find $\langle a^{26} \rangle$, $\langle a^{17} \rangle$, $\langle a^{18} \rangle$ and $|a^{26}|$, $|a^{17}|$ and $|a^{18}|$.

(c) Find the order of each element of U(15).

3. (a) Let $\alpha = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 2 & 3 & 4 & 5 & 1 & 7 & 8 & 6 \end{bmatrix}$ and

$$\beta = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 1 & 3 & 8 & 7 & 6 & 5 & 2 & 4 \end{bmatrix}$$

Write α, β and αβ as a product of disjoint cycles. Also find β^{-1}.

(b) Construct a complete Cayley table for D_4, the group of symmetries of a square. Is D_4 Abelian? Justify.

(c) Find all the left cosets of $\{1,15\}$ in $U(32)$.

4. (a) Let $H = \left\{\begin{bmatrix} a & b \\ 0 & d \end{bmatrix} : a, b, d \in \mathbb{R}, ad \neq 0\right\}$. Is H a normal subgroup of $GL(2, \mathbb{R})$? Justify.

(b) State Lagrange's theorem for finite groups. Prove that in a finite group, the order of each element of the group divides the order of the group.

(c) Let G be a group of permutations. For each σ in G, define

$$sgn(\sigma) = \begin{cases} 1 & \text{if } \sigma \text{ is an even permutation,} \\ -1 & \text{if } \sigma \text{ is an odd permutation.} \end{cases}$$

Prove that the function sgn is a homomorphism from G to the multiplicative group $\{-1,1\}$. What is the Kernel?

5. (a) (i) Describe all the subrings of the ring of integers.

(ii) Let a belong to a ring R. Let $S = \{x \in R \mid ax = 0\}$. Show that S is a subring of R.

(b) Prove that a finite Integral Domain is a field. Hence, show that $\mathbb{Z}_p$ is a field, where p is a prime number.

(c) State and prove the subring test.

Let $R = \left\{ \begin{bmatrix} a & a-b \\ a-b & b \end{bmatrix} \mid a, b \in \mathbb{Z} \right\} \subseteq M_2(\mathbb{Z})$ where $M_2(\mathbb{Z})$ is the ring of 2×2 matrices over 2. Prove or disprove that R is a subring of $M_2(\mathbb{Z})$.

6. (a) Define an ideal of a ring R. State the ideal test. Hence, prove that $n\mathbb{Z}$ is an ideal of $\mathbb{Z}$.

(b) Let $f : R \to S$ be a ring homomorphism. Prove that

(i) f(A) is a subring of S where A is a subring of R.

(ii) $f^{-1}(B) = \{r \in R \mid f(r) \in B\}$ is an ideal of R where B is an ideal of S.

(c) Determine all the ring homomorphisms from $\mathbb{Z}_{12}$ to $\mathbb{Z}_{30}$.

[This question paper contains 4 printed pages.]

Your Roll No..............

Sr. No. of Question Paper : 4014 **H**

Unique Paper Code : 2352572401

Name of the Paper : Abstract Algebra

Name of the Course : **B.A./B.Sc. (Programme) with Mathematics as Non-Major/ Minor – DSC**

Semester : IV

Duration : 3 Hours Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.

2. Attempt **all** questions by selecting **two** parts from each question.

3. Each part carries **7.5** marks.

4. Use of Calculator not allowed.

1. (a) Let n be a fixed positive integer greater than 1. If a mod n = a' and b mod n = b', then prove that (a + b)mod n = (a' + b')mod n and (ab)mod n = (a'b')mod n.

(b) Define a Group. Show that the set {5,15, 25, 35} is a group under multiplication modulo 40. What is the identity element of this group? Is there any relationship between this group and U(8)?

(c) Show that G = GL(2, ℝ), group of 2×2 matrices with real entries and nonzero determinants is non abelian. Also, construct a Cayley table for U(12).

2. (a) Let G be a group and H a nonempty subset of G. Prove that if ab^{-1} is in H whenever a and b are in H, then His a subgroup of G. Also, find the order of 7 in U(15).

(b) Let G be a group and let $a \in G$. Prove that a and a^{-1} have the same order. Illustrate the above result in the group $\mathbb{Z}_{10}$.

(c) Let a be an element of order n in a group G and let k be a positive integer. Show that

$$\langle a^k \rangle = \langle a^{\gcd(n,k)} \rangle \text{ and } |a^k| = \frac{n}{\gcd(n,k)}.$$

3. (a) If $\alpha = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 4 & 5 & 1 & 3 \end{bmatrix}$ and $\beta = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 1 & 4 & 3 & 5 \end{bmatrix}$.

Show that $\alpha\beta \neq \beta\alpha$. Also compute the value of and find order of $\beta^{-1}\alpha\beta$.

(b) Construct a complete Cayley table for D_4, the group of symmetries of a square. Find the inverse of each of the element in D_4.

(c) Let H = {(1), (1 2)(3 4), (1 3)(2 4), (1 4)(2 3)}. Find all the left cosets of H in A_4, the alternating group of degree 4.

4. (a) State Lagrange's theorem for finite groups. Show that a group of prime order is cyclic.

(b) State the normal subgroup test. Prove that SL(2, $\mathbb{R}$), the group of 2×2 matrices with determinant 1 is a normal subgroup of GL(2, $\mathbb{R}$), the group of 2×2 matrices with non zero determinant.

(c) Let f: G $\rightarrow$ G' be a group homomorphism. Prove that if H be a cyclic subgroup of G then f(H) is a cyclic subgroup of G'. Prove or disprove that the map g: ($\mathbb{R}$, +) $\rightarrow$ ($\mathbb{R}$, +) given by $g(x) = x^2$ is a homomorphism.

5. (a) (i) Prove that the set $A = \left\{ \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix} \middle| a, b \in \mathbb{Z} \right\}$ is a subring of the ring of all 2 × 2 matrices over $\mathbb{Z}$.

(ii) Prove that the ring $\mathbb{Z}_p$ of integers modulo p where p is a prime, is an integral domain.

(b) Define characteristic of a ring. Let R be a ring with unity 1. Prove that if 1 has infinite order under addition then charR = 0. If 1 has order n under addition, then charR = n.

(c) Prove that intersection of any collection of subrings of a ring R is a subring of R. Does the result hold for the union of subrings? Justify.

6. (a) State the ideal test. Let $\mathbb{R}[x]$ denote the set of all polynomials with real coefficients. Let $A = \{p(x) \in \mathbb{R}[x] \mid p(0) = 0\}$. Prove that A is a principal ideal of $\mathbb{R}[x]$.

(b) Determine all the ring homomorphisms from $\mathbb{Z}_n$ to itself.

(c) State and prove the First Isomorphism Theorem for rings.

[This question paper contains 12 printed pages.]

Your Roll No...............

Sr. No. of Question Paper : 4999 **H**

Unique Paper Code : 2352202402

Name of the Paper : Introduction to Graph Theory

Name of the Course : **B.A. (Programme) with Mathematics as Major (DSC-2)**

Semester : IV

Duration : 3 Hours Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.

2. Attempt **all** questions by selecting **two** parts from each question.

3. Both parts of a question to be attempted together.

4. **All** questions cany equal marks.

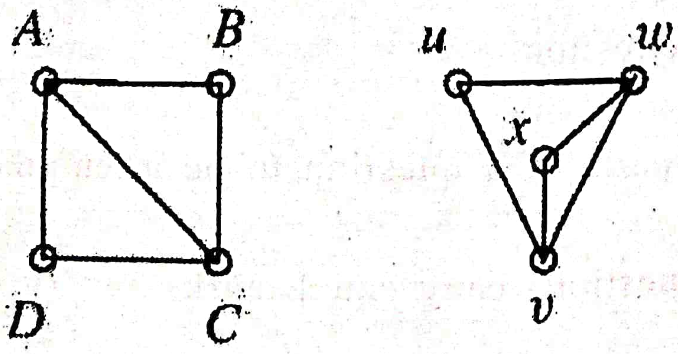
1. (a) Define Eulerian graph. Write the degree sequence of $K_{3,5}$. Is it bipartite? Justify. (7.5)

(b) Why can there not exist a graph whose degree sequence is 5, 4, 4, 3, 2, 1? What is degree of each vertex in the graph $K_{m,n}$. Define Adjacency matrix of a graph. Draw the graph whose

adjacency matrix is given by $\begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix}$.

(7.5)

(c) When are two graphs said to be isomorphic. Are the following two graphs isomorphic. If yes, find an isomorphism. (7.5)



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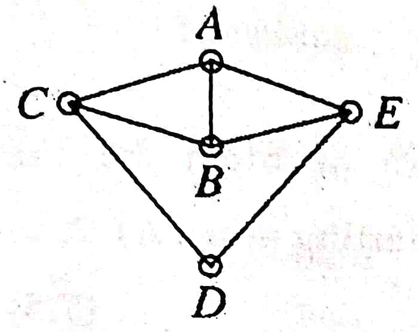
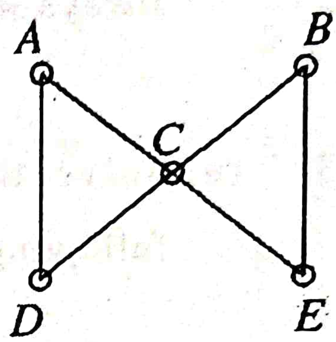
2\. (a) Define Hamiltonian cycle. Are the following two graphs Hamiltonian? Justify (7.5)

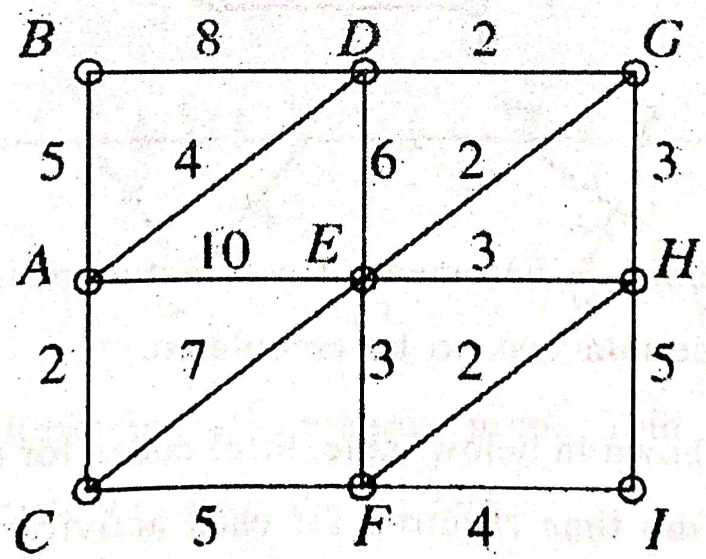


$\mathcal{G}_1$



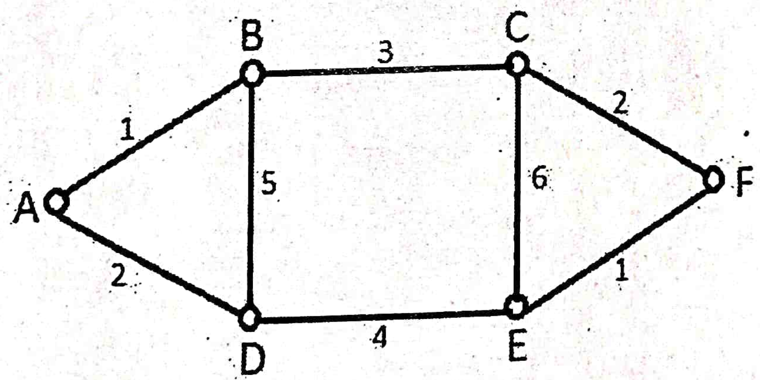
$\mathcal{G}_2$

(b) Use DIJKSTRA'S Algorithm to find shortest path from vertex A to all other vertices in a weighted graph. (7.5)



(c) At most social function, there is a lot of handshaking. Prove that the number of people who shake the hands of an odd number of people is always even. (7.5)

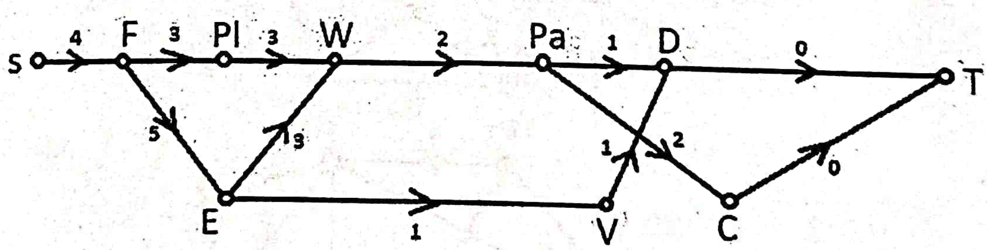
3. (a) Solve the Chinese postman problem for the following weighted graph (starting from A). (7.5)



(b) To finish a basement, a contractor must arrange for certain task to be completed.

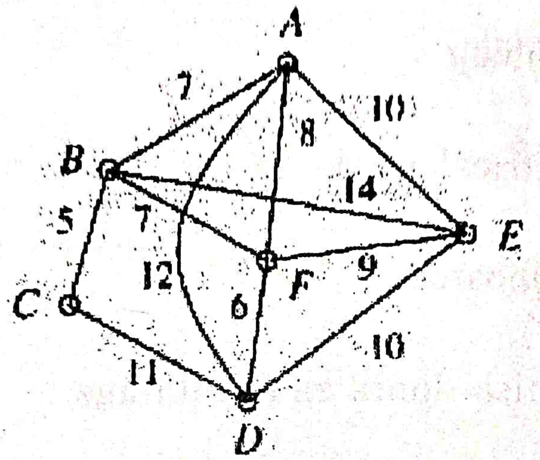
As shown in below table, brief codes for reference and the time required for each activity. (7.5)

Task	Code	Time
Floor installation	F	4
Plumbing	Pl	3
Electrical work	E	5
Wallboard	W	3
Varnish doors and moldings	V	1
Paint	Pa	2
Install doors and moldings	D	1
Lay Carpet	C	2

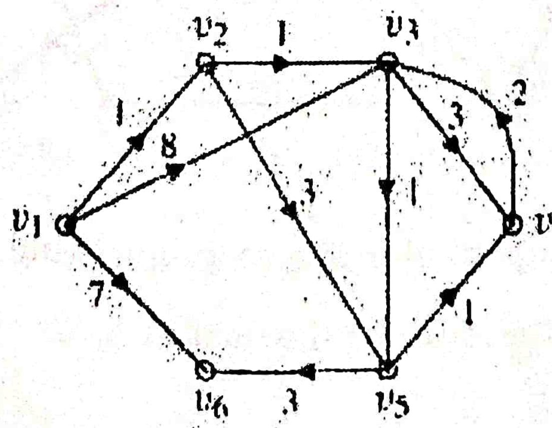


Referring to the above graph, find the minimum time to complete the task.

(c) Find the minimum spanning tree for the following graph (starting with the vertex A) (7.5)



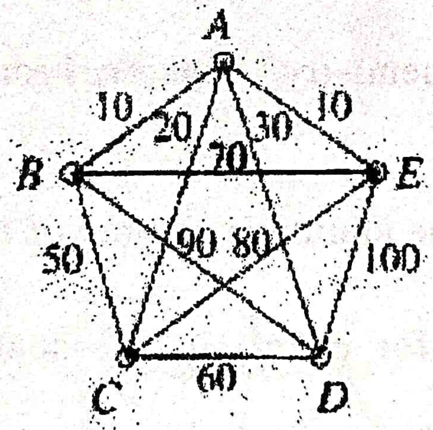
4. (a) Apply Bellman's algorithm to find the lengths of shortest paths from v_1 to each of other vertices. (7.5)



(b) Find a permutation matrix P such that $A_2 = PA_1P^T$ (7.5)

$$A_1 = \begin{bmatrix} 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \quad A_2 = \begin{bmatrix} 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

(c) Solve the Travelling Salesman's Problem for the following graph. (Start at A) (7.5)



5. (a) Let $\Delta(G)$ be the maximum of the degrees of the vertices of a graph G. Then $X(G) \leq 1 + \Delta(C)$. (7.5)

(b) Suppose that in one particular semester there are students taking each of the following combinations of courses.

(i) Mathematics, Physics, French.

(ii) Mathematics, English, German.

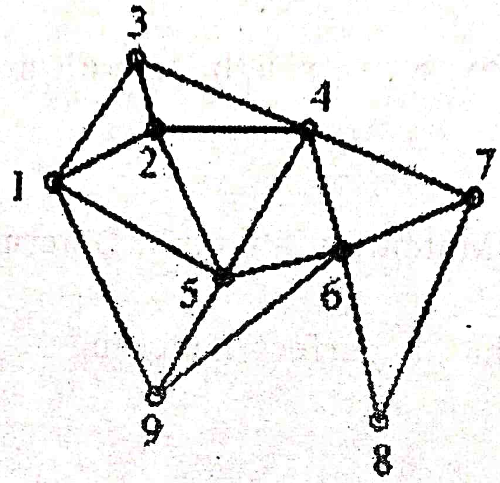
(iii) English, French, Physics.

(iv) Chemistry, Physics, French.

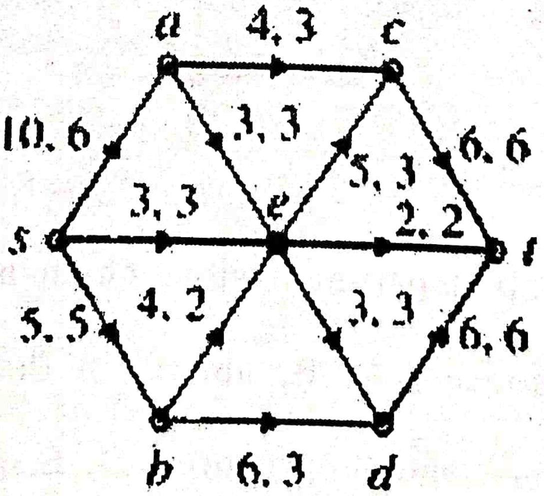
What is the minimum number of examination period required for exams in the specified courses so that students involved have no conflicts? Find a possible schedule that uses this minimum number of periods. (7.5)

(c) Define coloring of graph. What is Chromatic Number? Find Chromatic Number of the graph.

(7.5)



6. (a) For the network shown : (7.5)

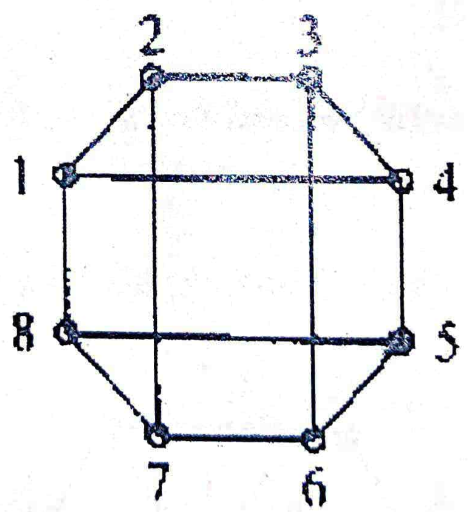


(i) Verify the law of conservation of flow at

and

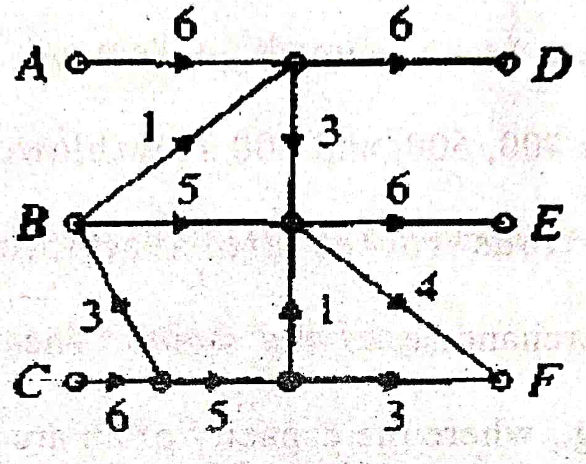
(ii) Find the capacity of the (s,t)-cut defined by S = {s, a, b, c, d, e} and T = {t}.

(b) Define Matching in Graph. Determine whether the graph has a perfect matching. (7.5)



(c) A small department store chain maintains three warehouses, A, B, and C in the south of the province and three stores, D, E, and F, in rural communities in the far north. The warehouses have, respectively, 500, 500, and 900 snow blowers in stock on October 1. The first blizzard of winter is forecast to start near midnight, October 2, and there is an immediate demand from the stores for 700, 600, and 600 snow blowers, respectively. Various routes are available for shipping merchandise to the stores. These are shown in Fig., where the capacity of an arc uv is the largest number of snow blowers that can be shipped from u to v in the course of a single day.

The vertices in the middle should be thought of as middlemen, baggage handlers, for example, who can only cope with so many snow blowers at once. We will assume that freight can cover as many arcs as necessary in a single day. Can all the required snow blowers reach the stores before the blizzard arrives? If not, how close can the company come to satisfying demand? (7.5)



[This question paper contains 4 printed pages.]

Your Roll No..............

Sr. No. of Question Paper : 2240 **H**

Unique Paper Code : 62357604

Name of the Paper : Differential Equations

Name of the Course : **B.A. (Prog.) – DSE**

Semester : VI

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.

2. Attempt any **two** parts from each question.

3. **All** questions are compulsory.

1. (i) Solve the differentials equation

$$(x^2 + y^2 + 1)\, dx - 2xy\, dy = 0$$

(ii) Solve

$$y + px = p^2x^4$$

P.T.O.

(iii) Find the solution

$$y = 2px + y^2p^3$$

2. (i) Find the general solution of

$$(D^2 + 4)y = \text{Sin } 3x + e^x$$

(ii) Solve

$$x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} - y = x^3$$

(iii) Solve the following equation

$$\frac{dy}{dx} - 7x + y = 0, \quad \frac{dy}{dt} - 2x - 5y = 0$$

3. (i) Using the variation of parameters, solve

$$y'' + 4y = \sin x$$

(ii) Solve $(y + z)dx + (z + x)dy + (x + y)dz = 0$.

(iii) Show that the e^{2x} and e^{3x} are linearly independent solution of $y'' - 5y + 6y = 0$. Find the solution $y(x)$ with the property that $y(0) = 0$ and $y'(0) = 1$.

4. (i) Form the partial differential equation of the equation $z = a(x + y) + b$.

(ii) Find the general solution of the equation $(y^2z/x)p + xzq = y^2$.

(iii) Find the complete integral of $x^2 p^2 + y^2 q^2 = z^2$.

5. (i) Form the partial differential equation of the equation $z = f(x^2 - y^2)$.

(ii) Find the general solution of the equation $p + 3q = z + \cot(y - 3x)$.

(iii) Find the complete integral of $z^2 = pqxy$.

6. (i) Find the differential equation of all spheres of radius, having center in the xy–plane.

(ii) Find the general solution of the equation $x(y^2 + z)p - y(x^2 + z)q = z(x^2 - y^2)$.

(iii) Find the complete integral of $p + q = pq$.

[This question paper contains 8 printed pages.]

Your Roll No...............

Sr. No. of Question Paper : 3473 **H**

Unique Paper Code : 42357602

Name of the Paper : DSE – Probability and Statistics

Name of the Course : **CBCS-LOCF: B.Sc. Physical Sciences / Mathematical Sciences**

Semester : VI

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.

2. Attempt all the **six** questions.

3. Each question has **three** parts. Attempt any **two** parts from each question.

4. Each part in Question **1, 3, 5** carries **6** marks.

5. Each part in Question **2, 4, 6** carries **6.5** marks.

6. Use of scientific calculator is allowed.

P.T.O.

1. (a) Let $\{C_n\}$ be an arbitrary sequence of events. Show that

$$P\left(\bigcup_{n=1}^{\infty} C_n\right) \leq \sum_{n=1}^{\infty} P(C_n).$$

(b) Find a formula for the probability distribution of the total number of heads obtained in six tosses of a balanced coin.

(c) The pdf of the random variable X is given by

$$f(x) = \begin{cases} \dfrac{c}{\sqrt{x}} & ; \ 0 < x < 4 \\ 0 & ; \ \text{elsewhere} \end{cases}$$

Find :

(i) The value of c.

(ii) $P\left(X < \frac{1}{4}\right)$ and $P(X > 1)$.

2. (a) Find the moment generating function of the random variable whose probability density is given by

$$f(x)=\begin{cases} e^{-x} & ; \ x>0 \\ 0 & ; \ \text{elsewhere} \end{cases}$$

Use it to find an expression for μ_r', where $\mu_r' = E[X^r]$ is the rth moment about the origin.

(b) Define geometric distribution. If the probability is 0.75 that an applicant for a driver's license will pass the road test on any given try, what is the probability that an applicant will finally pass the test on the fourth try?

(c) Show that Poisson distribution is a limiting case of binomial distribution when $n \rightarrow \infty$, $\theta \rightarrow 0$, while $n\theta = \lambda$ remains a constant.

P.T.O.

3. (a) Show that if a random variable has a uniform density with the parameters α and β, the probability that it will take on a value less than $\alpha + p(\beta - \alpha)$ is equal to p.

(b) Let X and Y be two random variables with joint probability mass function :

$P(x,y)=\frac{x+y}{12}$; for x = 1, 2 and y = 1, 2, and zero elsewhere.

Show that $E[XY] \neq E[X]E[Y]$.

(c) Let X and Y be discrete random variables having joint probability mass function-

$$P(x,y)=\begin{cases}\left(\frac{1}{2}\right)^{x+y} & ;\ 1\le x<\infty,\ 1\le y<\infty,\ x, y\in Z\\ 0 & ;\ \text{elsewhere}\end{cases}$$

Determine the joint moment generating function of X and Y.

4. (a) Let X_1 and X_2 have the joint pdf:

$$f(x_1, x_2) = \begin{cases} x_1 + x_2 \;; & 0 < x_1 < 1,\ 0 < x_2 < 1 \\ 0 \;; & \text{otherwise} \end{cases}$$

(i) Find the marginal pdf of X_1, X_2.

(ii) Find the probability $P\left(X_1 \le \frac{1}{2}\right)$.

(iii) Find the probability $P(X_1 + X_2 \le 1)$.

(b) Let (X, Y) be a discrete random variable having joint probability mass function :

$$P(x, y) = \begin{cases} \dfrac{x + 2y}{18} \;; & (x,y) = (1,1), (1,2), (2,1), (2,2) \\ 0 \;; & \text{elsewhere} \end{cases}$$

Determine the conditional mean and variance of Y given $X = x$ for $x = 1$.

(c) Let X and Y have the joint pdf :

$$f(x,y)=\begin{cases}6y & ; \ 0<y<x<1 \\ 0 & ; \ \text{elsewhere}\end{cases}$$

Show that E[Y] = E[E[Y|X]].

5. (a) Let

$$P(x,y)=\begin{cases}\dfrac{1}{16}, & \text{if } x=1,2,3,4:\ y=1,2,3,4 \\ 0, & \text{otherwise}\end{cases}$$

be the joint probability mass function of X and Y. Show that X and Y are independent.

(b) The joint density of two random variables X and Y is given as

$$f(x,y)=\begin{cases}x.\,e^{-x(1+y)}; & x>0 \text{ and } y>0 \\ 0 & ; \ \text{elsewhere}\end{cases}$$

Find the regression equation of Y on X.

(c) Using method of least square to fit a straight line for the following data:

X	0	1	2	3	4
Y	1	1.8	3.3	4.5	6.3

6. (a) Let X and Y have joint probability mass function described as follows :

(x,y)	$(1,1)$	$(1,2)$	$(1,3)$	$(2,1)$	$(2,2)$	$(2,3)$
$P(x,y)$	$\frac{2}{15}$	$\frac{4}{15}$	$\frac{3}{15}$	$\frac{1}{15}$	$\frac{1}{15}$	$\frac{4}{15}$

Find the correlation coefficient between X and Y.

(b) A soft drink vending machine is set so that the amount of drink dispensed is a random variable with a mean of 200 milliliters and a standard deviation of 15 milliliters. What is the probability that the average amount dispensed in a random sample of size 36 is at least 204 milliliters?

(c) Suppose the number of items produced in a factory during a week is a random variable with mean 500.

(i) What can be said about the probability that this week's production will be at least 1000?

(ii) If the variance of a week's production is known to equal 100, then what can be said about the probability that this week's production will be between 400 and 600?